Research Article

Mental Representations in Fraction Comparison

Holistic Versus Component-Based Strategies

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Abstract. In this study, we investigated the mental representations used in a fraction comparison task. Adults were asked to quickly and accurately pick the larger of two fractions presented on a computer screen and provide trial-by-trial reports of the types of strategies they used. We found that adults used a variety of strategies to compare fractions, ranging among just knowing the answer, using holistic knowledge of fractions to determine the answer, and using component-based procedures such as cross multiplication. Across all strategy types, regression analyses identified that reaction times were significantly predicted by numerical distance between fractions, indicating that the participants used a magnitude-based representation to compare the fraction magnitudes. In addition, a variant of the problem-size effect (e.g., Ashcraft, 1992) appeared, whereby reaction times were significantly predicted by the average cross product of the two fractions. This effect was primarily found for component-based strategies, indicating a role for strategy choice in the formation of mental representations of fractions.

Keywords: fractions, magnitude-based representation, distance effect, problem-size effect, holistic strategies, component-based strategies

Mental arithmetic computations, including those involving fractions, can be a significant part of daily cognitive efforts (Dehaene, 1992). People perform mental computations with fractions in many different contexts, ranging from determining the amount of gratuity to leave a restaurant server to estimating how much discount will be received as a result of a particular sale. Mental representations formed when working with fractions can be based on verbal descriptions of ratio ("one out of every two students is female") or part-whole visualizations of concrete objects such as lines or shapes (Kieren, 1988). Additionally, these representations may be consistent across cultures (Watanabe, 2006).

Despite the importance of having proficiency with fractions in daily life, a disturbing trend in the US has emerged; among 15 areas within mathematics, early high school students seem to have the weakest knowledge in the area of rational numbers and operations involving fractions and decimals (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). The problems with fractions are not limited to high school students: Adults also tend to perform poorly on problems that involve fractions and their related concepts. The National Assessment of Adult Literacy (NAAL) found that 22% of adults scored in the below basic level on the quantitative scale (involving simple judgments about decimals, probabilities, percentages, and frequencies), which represented the highest degree of illiteracy of the three types that were measured (Kutner et al., 2007).

Given the importance of fractions in everyday life and the disturbing national decline in adults' fraction proficiency, relatively few studies have attempted to understand the cognitive mechanisms underlying fraction computation. Furthermore, the evidence from the few studies examining this issue has been equivocal. For example, Bonato, Fabbri, Umiltá, and Zorzi (2007) investigated the numerical distance effect, which refers to the tendency for numbers that are far apart on the number line to be compared more quickly than numbers that are closer together (Moyer & Landauer, 1967). When used with numerical stimuli, the distance effect is commonly taken as evidence for the activation of a magnitude-based representation (Dehaene, Dehaene-Lambertz, & Cohen, 1998). Bonato et al. found that response times (RTs) were better predicted by the distance between numerators (or denominators) than by the actual numerical distance between the fractions. The authors concluded that participants used comparison strategies that processed the componential information in the fractions rather than accessing a magnitude-based representation. This conclusion, however, may be questioned due to the choice of stimuli Bonato et al. used. In their Experiments 1 and 2, participants were presented with unit fractions (fractions of the form $1/n$) and asked to compare the given fraction to the standard 1/5. Given that only the denominators varied in these fractions, it is not surprising that the difference between denominators was the best predictor of performance, hence leading Bonato et al. to conclude that people only attend to the components of the fraction rather than forming a magnitude-based representation.

In contrast to Bonato et al. (2007), Schneider and Siegler (2010) found that adults do use magnitude-based representations of fractions by exhibiting large distance effects in a fraction comparison task. Schneider and Siegler used stimulus sets that included non-unit fractions, resulting in lower

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correlations among the numerators, denominators, and fraction values. With these stimuli, compared to those used in Bonato et al. (2007), it should have been more difficult for participants to devise component-based strategies. Indeed, while Schneider and Siegler found a significant negative correlation between numerical distance and performance (thus demonstrating the distance effect), they found a small and nonsignificant correlation between the values of numerator and denominator and performance.

Similarly, Meert, Grégoire, and Noël (2009) found that adults tend to process componential information when comparing fractions with common denominators, and tend to use magnitude-based representations when comparing fractions with common numerators. Meert et al. concluded that adults process both types of information depending on the properties of the fractions being compared, which suggests that adults may process fraction information according to their chosen comparison strategy. However, a more thorough account of these strategies and the types of mental representations that they activate is currently lacking.

None of the studies discussed earlier specifically analyzed the types of strategies that were used by participants to compare fraction magnitudes. This is likely an important piece to the puzzle. Some types of component-based strategies, by their very nature, would be unlikely to result in a magnitude-based representation. For example, one method that is commonly taught in the US for comparing fractions is the cross multiplication algorithm (Boston, Smith, & Hillen, 2003), where two fractions *a/b* and *c/d* are compared by comparing the products of opposite numerators and denominators; that is, the products *ad* and *bc*. Since one would only have to perform two multiplications of cross products in order to compare the fraction magnitudes, there is no a priori reason to believe that a representation of the overall magnitude of the fractions would be formed. On the other hand, when participants use a holistic strategy, such as benchmarking the fraction values to other known fractions (Reys, 1999), magnitude-based representations may be activated, either through the mathematical properties that are used or the visualization of a mental number line. For example, one could correctly judge that 4/5 is bigger than 1/3 since 4/5 is bigger than 1/2, and 1/2 is bigger than 1/3. In this case, the use of the transitive property of order combined with a notion of the relative position of 1/2 with each fraction combines to produce a correct answer based on numerical concepts alone.

In the present study, we asked participants to make speeded magnitude comparisons of pairs of fractions while providing verbal self-reports of the strategies they used to make each comparison. This method models the procedure used by LeFevre, Sadesky, and Bisanz (1996) to analyze the strategies used by adults in simple arithmetic. To analyze the types of mental representations that were formed in using these strategies, we investigated two classic effects in numerical cognition. The numerical distance effect (Moyer & Landauer, 1967) was used to test whether participants accessed magnitude-based representations of the fractions being compared. If participants access the magnitude of each fraction before comparing them, their reaction times should vary as a function of the distance between the two fractions being compared. Specifically, participants should respond more slowly to fraction pairs that have a small numerical distance between them than fraction pairs that have a large numerical distance between them. Alternatively, if participants only attend to the components of the fractions rather than the magnitudes of the fractions, the numerical distance between the fractions should have little influence on reaction times.

The problem-size effect (e.g., Ashcraft, 1992) was used to investigate the extent to which componential representations were used to compare fraction magnitudes. The problem-size effect is a classic phenomenon in numerical cognition in which mathematical computations (addition or multiplication) are faster for smaller problems than for larger problems. If participants rely on computations with the components of a pair of fractions (such as is the case in cross multiplication), the problem-size effect should predict an increase in reaction time as the average cross product of the fraction pair increases. If, on the other hand, participants do not rely on computations with the fraction components, there should be little influence of average cross product on reaction times.

Method

Participants

Twenty-nine undergraduate students at Texas A&M University – Commerce participated in the present experiment (22 female; 26 right-handed). The mean age of the participants was 27.1 years (range 18–55 years; median 25 years; standard deviation 8.34 years). The participants were volunteers from freshman-level mathematics courses who participated for partial course credit.

Stimuli and Procedure

The stimulus set consisted of a set of 48 fraction pairs (see Appendix). The fraction pairs were composed of non-equivalent, reduced proper fractions (each with numerical magnitude less than 1) with numerators and denominators less than 10. The set of 48 fraction pairs was composed of three sets of 16 pairs, with each set representing one of three critical comparison fractions; 1/2, 1/3, or 2/3. This choice was made to prompt a variety of strategies, as piloting indicated that comparisons with 1/2 tended to prompt different strategies and reaction times than comparisons with other fractions. In each group of 16, half of the fractions were less than the critical fraction, whereas the other half were greater than the critical fraction. Also, the left/right position of the larger fraction was equally distributed within the pairs. No fraction pairs were repeated, but each individual fraction was repeated exactly twice (in different left/right positions) in comparison with two different critical fractions.

For each of the 48 fraction pairs, we computed the numerical distance between the two fractions (as a decimal) and the average of the cross products. Across all fraction pairs, numerical distance ranged from 0.042 to 0.389 with a mean of 0.208 and standard deviation 0.104. Average cross products ranged from 3 to 21 with a mean of 9.38 and standard deviation 4.53. Minimal correlation was found between the values of numerical distance and average cross product ($r = -0.19$).

Participants were tested individually in a quiet room during a single 45-min session. The fraction stimuli were presented using the PsyScopeX software package (build 53) on a 2.4 GHz MacBook Pro connected to an external 19-inch Dell LCD display. The viewing distance was 60 cm. Fraction stimuli were presented horizontally, 3.8° away from each other and 1.9° from the center of the screen. The height and width of each fraction were 6.7° and 2.9°, respectively. Arabic symbols were printed in black on a white background using Times New Roman font. All responses were collected using an IoLab Systems USB Response Box, on which participants initiated response options by pressing a specified button. The response box time-stamped the instance of any button press; since all timing was hardware-based, RTs were recorded with an accuracy of ±1 ms.

During the instruction phase of the experiment, participants were told that the purpose of the experiment was to investigate the different strategies used to compare simple fractions. A set of examples (e.g., LeFevre et al., 1996) was presented (see Table 1). At the conclusion of this set of examples, the experimenter asked each participant if he/she understood each strategy, and if not, further explanation was given to the participant.

Participants were told that each trial would consist of two parts. During the first part, they would see a fraction pair appear on the screen. Using the response box in front of them, they were to press the button corresponding to the larger fraction as quickly and accurately as possible. Only two buttons on the button box were available for use; if the fraction presented on the left side of the screen was larger, the left button was pressed on the button box. If, on the other hand, the fraction presented on the right side of the screen is larger, the right button was pressed.

For the second part, the participant was asked to tell the experimenter how they solved the problem, using as much detail as possible. The experimenter recorded the responses using a lapel microphone routed through a mixer to the computer's line-in port and captured the audio with the Audacity software package. Three practice problems were then presented, and participants were reminded to answer as quickly and accurately as possible. The three practice problems were the pairs [1/2 1/3], [1/2 2/3], and [1/3 2/3]; these pairs were not repeated during the testing block. Feedback in the form of an audible beep (for correct answers) and an audible buzz (for incorrect answers) was presented during the practice problems. After the practice problems were completed, the participants were given a chance to ask any further questions before proceeding to the testing phase.

During the testing phase, each trial began with the sentence, "Say 'Go' when ready," presented in the center of the screen. When the participant's voice triggered the experiment software, one of the randomly selected fraction pairs appeared at the center of the screen. The fraction pair remained on the screen until either a button was pressed or 15 s had elapsed. No feedback was given during the testing phase. Following the button press, the sentence, "Describe how you figured out your answer," appeared at the center of the screen, prompting the participant to describe his/her approach. The only interaction between the experimenter and participant was to ask clarifying questions, such as, "Explain your strategy again." Once the experimenter recorded the participant's response, the experimenter initiated the next trial.

After finishing the testing phase, participants completed the Addition test and the Subtraction-Multiplication test from the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Dermen, 1976). The Addition test was composed of two pages of three-addend addition problems (for a total of 120 problems). The Subtraction-Multiplication test consisted of two pages of two-digit subtraction problems and two-by-one digit multiplication problems (for a total of 120 problems). Participants were allowed 2 min per page to correctly answer as many problems as they could. Arithmetic fluency was defined as the total number of correct answers on both tests.

Results

Accuracy

A total of 1,392 trials were administered. Of these trials, 30 trials were discarded due to either a failure of the experimental apparatus or a failure to respond to a fraction pair within the timeout period of 15 s. Of the remaining 1,362 trials, 86.7% were answered correctly. A summary of results by item can be found in the Appendix.

Table 1. Example fraction strategies that were randomly presented in the instruction phase

Strategy type	Instruction
Just knew it	You may just remember that 1/2 is greater than 1/4. The answer just pops into your head.
Common denominators	You may get common denominators and compare 2/4 to 1/4.
Visualization	You could also visualize 1/2 and 1/4 in terms of something like a pizza or a number line and figure out which one is bigger that way.
Cross multiplication	You could use cross multiplication to find your answer.
Other	You could use some other strategy to figure it out.

Fraction-Task Performance

The overall fraction-task performance (measured by median RT and error rate) was analyzed via a repeated-measures multivariate analysis of variance, with critical fraction (1/3, 1/2, or 2/3) as a within-subjects factor. There was an overall main effect of critical fraction, $F(4, 25) = 15.93$, partial $\eta^2 = 0.815$, $p < .001$. Univariate tests confirmed that this was due to the significant effects of both median RT, $F(2, 56) = 35.15$, partial $\eta^2 = 0.557$, $p < .001$, and error rate, $F(2, 56) = 18.54$, partial $\eta^2 = 0.398$, $p < .001$. Participants tended to be faster and less error-prone when comparing fractions involving 1/2 (2,761 ms; 3.47% error) compared to fractions involving 1/3 (4,219 ms; 14.55% error) or 2/3 (5,130 ms; 15.45% error). Planned contrasts among critical fractions revealed a difference in median RT between fraction pairs involving 1/2 versus fraction pairs involving 1/3, $F(1, 28) = 38.53$, partial $\eta^2 = 0.579$, $p < .001$, and a difference in median RT between fraction pairs involving 1/3 and fraction pairs involving 2/3, $F(1, 28) = 13.32$, partial $\eta^2 = 0.322$, $p < .001$. A difference in error rate was also observed between fraction pairs involving 1/2 versus fraction pairs involving 1/3, $F(1, 28) = 22.26$, partial $\eta^2 = 0.443$, $p < .001$, but there was no difference in error rate between fraction pairs involving 1/3 and 2/3 as their respective critical fractions, ($F < 1$).

Fraction Comparison Strategies

As shown in Table 2, participants used a variety of strategies to compare the sizes of two fractions. Across all participants, five substantive strategies were repeatedly used. On 30.7% of the trials, participants reported just knowing the answer. Across all strategies, these strategies yielded the shortest reaction times (median RT = 1,999 ms) and moderate accuracy (9.1% error). The remaining four strategies represented a mix of holistic strategies and component-based strategies. Benchmarking (Reys, 1999) refers to the use of part-whole relationships to make decisions about the relative size of fractions. For example, 5/7 is greater than 1/3 because 5 is more than half of 7, but 1 is less than half of 3. Visualization, on the other hand, refers to the reported sole use of a visual model, such as a partitioned circle, to make judgments. Note that this strategy was particularly error-prone (error rate = 23.9%). Cross multiplication (Boston et al., 2003) involves computing the two cross products in a given fraction pair and using them to decide which fraction is larger. Two participants reported using cross multiplication on every trial. Although uncommon (only 5.5% of all trials), some participants used a decimal strategy that involved comparing two fractions by first converting each fraction to a decimal. Verbal reports from participants using the decimal strategy indicated that these decimal conversions occurred primarily through computational procedures such as scaling the components of the fraction to produce a denominator equal to 100.

The nature of the reported strategies allows a more broad classification into three distinct strategy types: (a) knowing the answer, (b) using a component-based strategy (such as cross multiplication or the decimal conversion strategy), or (c) using a holistic strategy (such as benchmarking or concrete visualization). To analyze how the types of strategies used are possibly related to numerical properties of the fraction stimuli, we computed the frequencies of trials on which participants used one of these three broad strategy categories as a function of numerical distance, average cross product, critical fraction, and same-numerator status. The results are presented in Table 3. For example, participants tended to report knowing the answer more often when the fraction pairs involved the critical fraction 1/2 or the fraction pairs contained the same numerator. Also, participants tended to use component-based strategies more frequently when fraction pairs were close together on the number line or did not involve the fraction 1/2. This relationship between strategy choice and numerical properties of the fraction pairs warrants a further analysis of how performance depends on these properties, which is presented below.

Structural Predictors of Performance

To further investigate the influence of participants' fraction comparison strategies on the distance effect and the problem-size effect, median RTs were analyzed by linear regression analyses to test whether performance varied with the numerical distance between the two fractions in a pair (e.g., 0.167 for 5/6 vs. 2/3) and/or the average cross product of the components in a fraction pair (e.g., 13.5 for 5/6 vs. 2/3, calculated as the average of 5×3 and 6×2). In addition, linear regressions were performed on three subsets of the data: pairs with 1/2 as the critical fraction, pairs not

Table 2. Types and performance of fraction comparison strategies

Strategy	Trials (%)	No. of people[a]	Frequency of use (%)[b]	Median RT (ms)	Error (%)
Just knew it	30.7	27	2–96	1,999	9.1
Cross multiplication	24.1	21	2–100	5,435	9.3
Benchmarking	19.6	20	2–54	3,286	9.4
Visualization	18.4	22	2–98	4,251	23.9
Decimals	5.5	6	2–92	5,209	8.0
Other	1.7	15	2–6	6,664	52.2

Note. [a]Number of participants who used the selected strategy at least once.
[b]The range of usage of a selected strategy over all participants who used the selected strategy at least once.

Table 3. Strategy distributions by distance, average cross product, critical fraction, and numerator (percentage in parentheses)

	Distance[a]		Average cross product[b]	
Strategy type	Close	Far	Small	Large
Know	129 (21.4)	289 (39.3)	336 (41.4)	82 (15.6)
Component-based	226 (37.4)	177 (24.1)	213 (26.2)	190 (36.1)
Holistic	249 (41.2)	269 (36.6)	263 (32.4)	255 (48.4)
	Critical fraction		Numerator	
Strategy type	Half	Non-half	Same numerator	Different numerator
Know	234 (51.0)	184 (20.9)	235 (63.5)	183 (18.9)
Component-based	83 (18.1)	320 (36.4)	65 (17.6)	338 (34.9)
Holistic	142 (30.9)	376 (42.7)	70 (18.9)	448 (46.2)

Note. [a]Close and far problems are defined according to whether the numerical distance between fractions is less than or greater than 0.2083, respectively.
[b]Small problems have average cross product less than or equal to 10, whereas large problems have average cross product greater than 10.

involving 1/2, and pairs with different numerators. The last of these analyses was run to investigate how the mental representations formed across all fraction pairs differed from those involving fraction pairs with different numerators. Specifically, we wanted to validate those strategy reports in which participants reported just knowing the answer. It is possible that for fraction pairs with the same numerator, participants could proceed by simply comparing the denominators, and thus not access the actual magnitudes of the fractions. Since this strategy doesn't necessarily have an accepted name, participants could have responded, "I just knew it." In this case, compared to all fraction pairs, one would expect the effect of distance on median RT to be different for those fraction pairs with different numerators, as such a strategy would not consistently work when both numerator and denominator differ. Separate analyses were also performed for the three different strategy types: know, component-based, and holistic (see Figure 1). Each regression analysis was performed by first identifying the significant predictors separately, then testing the relative contribution of each significant predictor in a multiple regression (if applicable).

Across all fractions and strategies, median RTs were significantly predicted by both overall distance and average cross product: respectively, $\beta = -0.69$, $t(46) = -6.54$, $p < .001$, and $\beta = 0.51$, $t(46) = 4.05$, $p < .001$ (see Table 4). When entered simultaneously in a multiple regression ($R^2 = 0.63$), overall distance and average cross product remained as significant predictors: respectively, $\beta = -0.62$, $t(45) = -6.70$, $p < .001$, and $\beta = 0.39$, $t(45) = 4.27$, $p < .001$. The same pattern was observed when the data set was reduced to only those fractions that did not involve 1/2 as a critical fraction. Median RTs were predicted by overall distance ($\beta = -0.43$, $t(30) = -2.61$, $p < .05$) and average cross product ($\beta = 0.37$, $t(30) = 2.18$, $p < .05$), and when entered simultaneously in a multiple regression ($R^2 = 0.36$), overall distance and average cross product remained as significant predictors: respectively, $\beta = -0.47$, $t(29) = -3.13$, $p < .01$, and $\beta = 0.42$, $t(29) = 2.77$, $p < .05$. On the other hand, neither predictor significantly predicted performance when the data set was reduced to those fractions involving 1/2 as the critical fraction, indicating that fraction comparisons involving 1/2 are processed differently than those not involving 1/2. In addition, when same-numerator pairs were excluded, we observed a pattern of predictors similar to that of the whole data set. For these fraction pairs, we found that the effect of distance on median RT was virtually identical to the distance effect for the whole stimulus set. This indicates that fraction pairs with different numerators were processed in a manner similar to the whole stimulus set, which helps to reduce the possibility that fraction pairs with the same numerator were somehow processed differently.

These results indicate that median RTs were predicted by numerical distance and average cross product, suggesting that participants formed both magnitude-based representations and component-based representations. To separate the contributions of these predictors, similar analyses were performed by strategy type. That is, for each analysis, median RTs were computed only for those trials where a specific strategy type was used, which included (1) just knew the answer, (2) used a component-based strategy, or (3) used a holistic strategy. A summary of these data can also be found in Table 4.

When participants reported just knowing the answer, the only significant predictor of median RT was overall distance, $\beta = -0.43$, $t(29) = -3.26$, $p < .01$. The same pattern was observed when the data set was reduced to only those fractions involving 1/2, $\beta = -0.55$, $t(10) = -2.49$, $p < .05$. Neither predictor was found to be significant when the data set was reduced to non-1/2 comparisons (p values were larger than 0.20 for both overall distance and average cross product). This indicates that when participants reported just knowing the answer, they relied mainly upon magnitude-based representations, especially when one of the fractions was 1/2. Also, since the effects of distance on median RT were virtually identical between the whole data set and the reduced data set including only fraction pairs with different

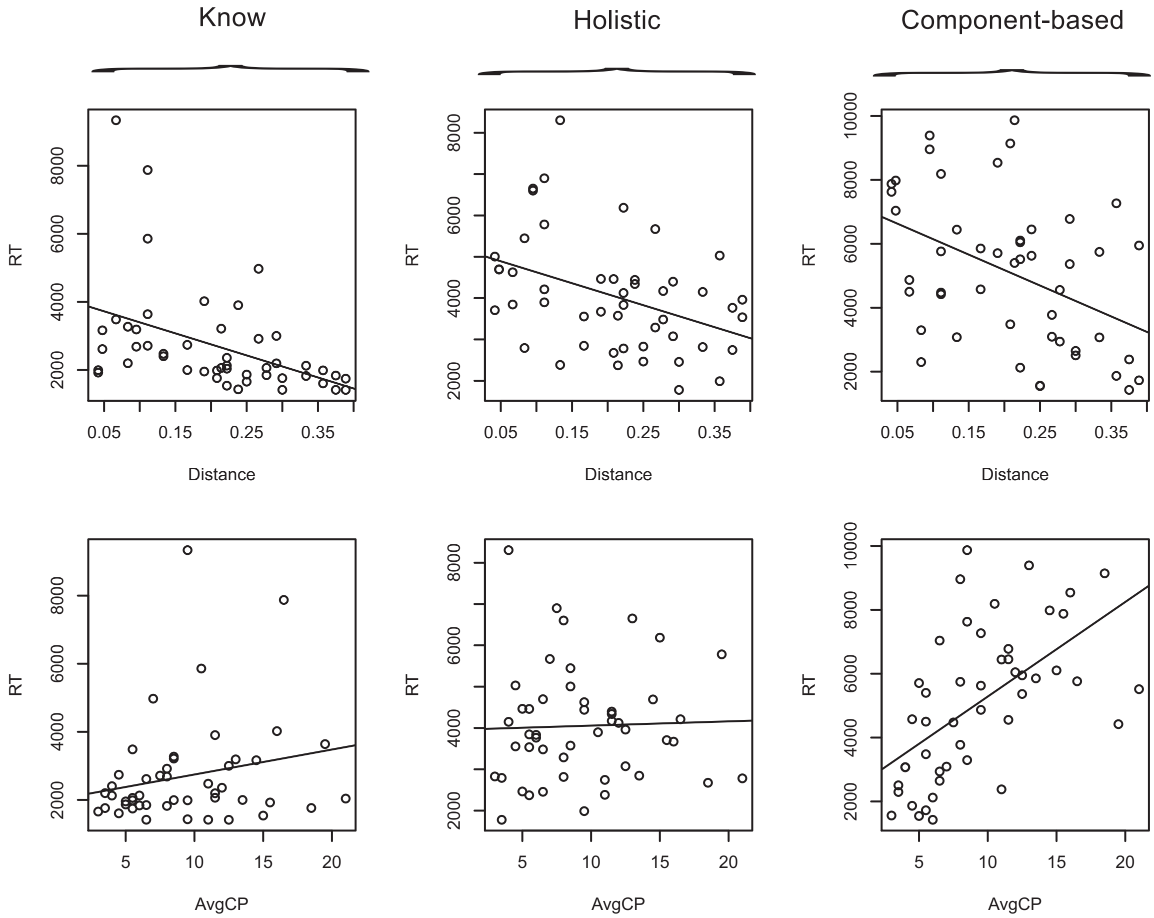

Figure 1. Median RT as a function of distance (first row) and average cross product (AvgCP, second row), grouped by strategy type. Regression lines have been added.

numerators, it is unlikely that same-numerator pairs were processed in a vastly different manner from different-numerator pairs.

In the cases when participants reported using a component-based strategy (cross multiplication or decimal conversion), median RTs were significantly predicted both by overall distance and average cross product: respectively, $\beta = -0.43$, $t(46) = -3.21$, $p < .01$, and $\beta = 0.57$, $t(46) = 4.72$, $p < .001$. When entered simultaneously into a multiple regression ($R^2 = 0.43$), both predictors remained

Table 4. Structural predictors of median RT, analyzed by strategy type and critical fraction

	All data[a]		Halves[b]		Non-halves[c]		Same-numerator pairs excluded[d]	
Strategy type	Distance	Average cross product	Distance	Average cross product	Distance	Average cross product	Distance	Average cross product
All	-0.69^{***}	0.51^{***}	-0.48	0.43	-0.43^{*}	0.37^{*}	-0.75^{***}	0.24
Know	-0.43^{**}	0.21	-0.55^{*}	0.01	-0.23	-0.02	-0.42^{*}	0.07
Component-based	-0.43^{**}	0.57^{***}	-0.26	0.54^{*}	-0.21	0.51^{*}	-0.36^{*}	0.34^{*}
Holistic	-0.40^{**}	0.03	0.28	0.12	-0.23	-0.22	-0.48^{**}	0.02

Note. All reported values are the standardized coefficients (β) in the linear regressions.
[a]The number of problems in the regressions was 48.
[b]The number of problems in the regressions was 12.
[c]The number of problems in the regressions was 36.
[d]All fraction stimuli with a same-numerator pair were removed prior to regressions. The number of remaining problems was 35.
$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

significant, with average cross product having a larger effect ($\beta = 0.51$, $t(45) = 4.43$, $p < .001$) than overall distance ($\beta = -0.33$, $t(45) = -2.89$, $p < .01$). A more telling picture of the data emerges when we restrict to the data subsets. Indeed, the only significant predictor of median RT for those problems involving 1/2 as a critical fraction is average cross product ($\beta = 0.54$, $t(10) = 2.40$, $p < .05$), whereas the effect of overall distance was not significant ($\beta = -0.26$, $t(10) = -1.01$, $p < .30$). A similar pattern was observed for those fractions that did not involve 1/2 as a critical fraction. Average cross product was a significant predictor of median RT ($\beta = 0.51$, $t(30) = 3.21$, $p < .01$) whereas overall distance was not a significant predictor of median RT ($\beta = -0.21$, $t(30) = -1.17$, $p < .25$). This data indicates that participants did not rely upon representations of magnitude when they used a component-based strategy. Whereas there was a small effect of overall distance in the aggregate data, it is interesting to note that this effect is absent in both subsets.

In the cases where participants reported using a holistic strategy (benchmarking or visualization), the only significant predictor of median RT was overall distance ($\beta = -0.40$, $t(46) = -2.96$, $p < .01$). In fact, average cross product was found to have virtually no effect on median RT ($\beta = 0.03$, $t(46) = 0.23$, $p > .80$). No significant predictors were found when we analyzed the subsets (all p values were larger than .20).

Discussion

The few previous studies examining the types of mental representations that are formed when people are engaged in a fraction comparison task have yielded conflicting results. However, the results of these studies have been based on indirect evidence. In contrast, we examined this issue by directly asking participants what type of strategy they had used when performing the comparison task and found that people do, in fact, represent magnitude in their mental representations of fractions. Our conclusions conflict with those of Bonato et al. (2007), but are consistent with those of Meert et al. (2009) and Schneider and Siegler (2010).

In addition, the present study examines fractions by bringing together the vast literature on the distance and problem-size effects with the vast literature on mathematical strategies, along with integrating their respective methodologies. To our knowledge, no studies have previously attempted this. We found that adults were able to successfully complete the fraction comparison task in a variety of ways. Trial-by-trial strategy reports revealed a variety of strategies that were employed across the fraction types, ranging from just knowing an answer to using a specific computational procedure. While the reliability and validity of self-reports can be questionable (Russo, Johnson, & Stephens, 1989), we found that performance measures (RT and error rate) varied in a predictable fashion across these reported strategies. Indeed, predicted effects such as the numerical distance effect and the problem-size effect appeared in the data, but each appeared in different ways according to which strategy was reported, indicating that aggregating these measures across strategy types can obscure important effects (Siegler, 1987).

The mental representations that are formed when people are engaged in a fraction comparison task seem to depend on two main factors: the nature of the fractions being compared and the type of strategy that is employed to perform the comparison. Fraction pairs involving 1/2 were compared much more quickly and accurately than fraction pairs that did not contain 1/2. This may be due to the fact that the fraction 1/2 enjoys a privileged representational status arising from repeated exposure to the fraction 1/2 over a lifetime of education. Indeed, 1/2 is one of the first fractions that children experience (Miller, 1984; Singer-Freeman & Goswami, 2001) and is easily visualized as one of two identical parts. Also, 1/2 is often used as a fraction benchmark, or "anchor," when performing fraction operations (Spinillo & Bryant, 1991). This repeated use of the half concept likely leads to a familiarity with 1/2 that is not achieved with other fractions.

The type of strategy used to compare fractions seems to be the most important factor in determining the type of mental representation that is formed. For all strategy types in the present study, median RT was predicted by the numerical distance between the two fractions. That is, as two fractions increase in their distance apart, the median RT on the fraction comparison task decreases. This replication of the numerical distance effect (Moyer & Landauer, 1967) provides evidence for a magnitude-based representation of fractions. In addition, the current results align with two recent imaging studies (Ischebeck, Schocke, & Delazer, 2009; Jacob & Nieder, 2009). In both studies, brain activation in the intraparietal sulcus, a region assumed to be specialized for magnitude representation, varied with the numerical distance between fractions, but not with the distance between individual components of the fractions, such as numerators or denominators. These data lend support to the idea that people do often encode the magnitude of fractions in fraction comparison tasks.

In light of this, it is still early to make strong conclusions about performance on fraction pairs in which participants reported just knowing the answer. Indeed, participants may often report just knowing the answer when they don't know what else to say. Two analyses were conducted to attempt to shed further light on this category. First, as noted in Table 3, the know response was used most often when fractions were far apart, had small components (on average), involved the critical fraction 1/2, and had the same numerator. This is important to note, as any future studies in which researchers wish to minimize this kind of response category should take these into consideration when designing stimulus sets.

We did note that in the "Know" category, the effect of distance on median RT was virtually the same for both the whole stimulus set and a reduced data set that removed all same-numerator pairs. This indicates that fraction pairs with different numerators were likely processed in a similar fashion to those with same numerators, for if they were not, it is unlikely that the overall distance effects would have remained the same when removing the same-numerator pairs. Critically, the presence of this distance effect (and the absence of any problem-size effect) lessens the probability that people simply compared the denominators when the numerators were the same. It is important to note, however, that this analysis is far from exhaustive, and future studies will need to be conducted in order to illuminate any strategy selection differences between such fraction stimulus characteristics.

The present study also investigated the influence of the problem-size effect (e.g., Ashcraft, 1992) on median RT. The problem-size effect refers to the tendency for small problems (such as 2×3) to be executed much faster than large problems (such as 8×9). We hypothesized that if people use a component-based strategy, such as cross multiplication, to process the components of fractions, the whole number computations inherent in this strategy would result in a form of the problem-size effect, whereby larger average cross products would result in longer median RTs. Although we found this to be the case, the effect was found only for the component-based strategies. To our knowledge, this is the only study that has analyzed the influence of problem size in a fraction context. Since the problem-size effect is a robust effect in whole number arithmetic, it stands to reason that computational strategies that involve the whole number components of fraction pairs would be subject to the problem-size effect as well. Extending this reasoning, it seems plausible that the problem-size effect could, in the future, be used as a valid measure of componential processing in fraction comparison.

It is also intriguing that, in studies involving the problem-size effect for whole number arithmetic computations, the effect on reaction time is rather small, and as such, it takes many trials for the effect to manifest (Zbrodoff & Logan, 2005). For purposes of comparison, note that in one classic study on the problem-size effect (Campbell & Xue, 2001), participants had to produce answers to 429 whole number arithmetic problems, and across all problems, the largest reaction time differential between large and small problems was 19 ms. In the current study, the problem-size effect (for component-based strategies) appeared with only 48 trials, and the reaction time penalty for large problems was much greater (6,459 ms for large problems, 4,209 ms for small problems, $F(1, 46) = 13.26, p < .001$). At present, it is not clear why this was the case.

One possibility for the large problem-size effect in the current study is that the measured reaction times did not independently assess both strategy selection and strategy execution. Many participants did not solely rely on one strategy for all fraction comparisons. Thus, the time from presentation of a fraction stimulus to the production of its answer likely contained a component of strategy selection as well as the strategy execution component. This complicates any interpretation of reaction time differences, especially any preliminary interpretation of the problem-size effect. It could be the case that the extra time present in large problems may stem from an increased difficulty in choosing the appropriate strategy when problems have larger components rather than in the computation itself. Future work that addresses this issue should take care to employ a design that would allow for independent analysis of these two components of strategic performance, such as the choice/no-choice method of Siegler and Lemaire (1997).

It should be noted that the reaction times in this study were quite long compared with other published studies on fraction comparison. In fact, the reaction times and error rates are closer to those reported in Experiment 3 of Schneider and Siegler (2010), where participants of limited skill were tested. In order to form a skill comparison between our participants and those of previous studies in mathematical cognition, we obtained arithmetic fluency scores for each participant. The mean arithmetic fluency score was 70.4, which is comparable to fluency scores in other math cognition studies (see, e.g., LeFevre et al., 1996). Unfortunately, none of the previously referenced studies on fractions computed fluency scores from which to make a comparison. One other possibility for explaining the long reaction times stems from the method of stimulus presentation in the current experiment. Since strategy reports were obtained, there was a definite pause between solutions of subsequent fraction comparison problems. These pauses could have contributed to a general slowing down across the fraction problems.

In conclusion, this study provides evidence that adults are generally successfully in encoding the magnitudes of fractions in a fraction comparison task, regardless of the type of strategy used to perform the fraction comparison. In addition, the problem-size effect emerges when component-based strategies such as cross multiplication are used, indicating that componential representations are also formed in some instances. These findings pose many new questions about adults' representations when they are engaged in fraction tasks. Future research should investigate how these different types of representations are formed and how they contribute to our overall understanding of how people think about fractions.

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Appendix

List of Experimental Pairs of Fractions

The median RT and the error rate by fraction pair (aggregated across all strategies)

Stimulus		RT (ms)	Error (%)	Stimulus		RT (ms)	Error (%)
4/5	2/3	3,200	6.9	1/5	1/2	1,778	0.0
5/6	2/3	3,810	6.9	1/6	1/2	2,174	0.0
8/9	2/3	4,006	6.9	1/8	1/2	1,837	6.9
7/8	2/3	3,205	6.9	2/7	1/2	2,372	10.3
2/3	7/9	4,926	20.7	1/2	1/4	1,657	0.0
2/3	5/7	6,119	37.9	1/2	1/7	1,685	6.9
2/3	6/7	3,930	17.2	1/2	1/9	1,746	10.3
2/3	3/4	5,174	20.7	1/2	2/9	2,771	13.8
2/5	2/3	3,613	24.1	4/7	1/3	4,440	34.5
3/5	2/3	4,868	34.5	5/9	1/3	5,076	13.8
4/9	2/3	5,821	17.2	3/7	1/3	6,399	27.6
5/8	2/3	5,575	27.6	3/8	1/3	6,243	44.8
2/3	3/8	3,251	17.2	1/3	2/5	4,410	31.0
2/3	4/7	7,168	27.6	1/3	5/8	4,038	20.7
2/3	5/9	5,762	24.1	1/3	3/5	4,997	10.3
2/3	3/7	4,305	17.2	1/3	4/9	4,021	20.7
3/4	1/2	1,981	3.4	1/4	1/3	2,366	13.8
5/7	1/2	4,121	13.8	1/7	1/3	1,999	3.4
6/7	1/2	2,102	10.3	1/9	1/3	2,129	10.3
7/9	1/2	2,767	13.8	2/9	1/3	4,932	13.8
1/2	4/5	2,101	10.3	1/3	1/5	3,098	3.4
1/2	5/6	2,680	6.9	1/3	1/8	1,986	13.8
1/2	7/8	2,098	10.3	1/3	2/7	4,697	20.7
1/2	8/9	2,239	3.4	1/3	1/6	2,754	10.3