INDIVIDUAL DIFFERENCES IN MENTAL REPRESENTATIONS OF FRACTION MAGNITUDE

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The numerical distance effect is a robust effect in mathematical cognition that describes a negative correlation of the numerical distance between two numbers and the time it takes to choose the larger number. The presence of this effect is commonly taken as evidence for a person's tendency to represent numbers conceptually on a mental number line; i.e, a magnitude-based representation.. In the current study, the size of the numerical distance effect decreased for individuals with high mathematics anxiety or high calculator use, indicating that those individuals tend to have less-developed magnitude-based representations of fractions.

Mathematical tasks induce people to form a wide range of mental representations of number. For example, when people are asked to quickly choose the larger of two numbers, they do so more quickly and accurately when the distance between the numbers is relatively large, compared to when the distance between the two numbers is small (e.g, Moyer & Landauer, 1967; Dehaene, 1992). Moreover, the response times tend to decrease as either a logarithmic function (Dehaene, Dupoux, & Mehler, 1990) or a linear function (Gallistel & Gelman, 2000) of the increasing distance between the two numbers. This is a robust effect in mathematical cognition known as the *numerical distance effect*, and its presence suggests that people use an analog magnitude-based representation (e.g., a mental number line) to compare natural numbers.

Recently, several researchers have begun to investigate the mental representations that people use when thinking about fractions. Bonato, Fabbri, Umiltà, and Zorzi (2007) had participants press a button to choose the larger of two fractions presented on a computer screen. They found that participants tended to compare the components of the fractions (numerators and denominators) and not the real numerical value (or magnitude) of the fractions. This led them to conclude that people did not form mental representations of fraction magnitude. In contrast, both Meert, Grégoire, and Noël (2009) and Schneider and Siegler (2010) found significant numerical distance effects in fraction comparison tasks, indicating that people do indeed form magnitude-based mental representations of fractions. Similarly, Faulkenberry and Pierce (2010) found that people exhibit a significant numerical distance effect regardless of the type of strategy (conceptual or procedural) employed to compare fractions, again indicating the presence of a magnitude-based fraction representation. However, the size of the distance effect (as measured by the coefficient of determination r^2) varied across the types of strategies used.

The current study investigated the influence of individual differences on magnitude-based representations of fractions. Affective variables are known to have significant effects on various aspects of mathematical cognition. Of particular interest to the current study are math anxiety (Ashcraft & Kirk, 2001), arithmetic skill (Campbell & Xue, 2001; LeFevre & Bisanz, 1986), and daily calculator use (Imbo & Vandierendonck, 2007; but see Campbell & Xue, 2001). All of the above-mentioned affective variables tend to negatively affect performance as measured by RT or error rates (or both). Given this, it is possible that having a detrimental level of one of these variables would result in a less-pronounced numerical distance effect when comparing fractions. That is, it is possible that these variables could be negatively associated with the successful use of the mental number line.

Participants were asked to make speeded judgments of fraction magnitude for simple proper fractions. For each participant, a measure of the size of the numerical distance effect was computed by regressing reaction time against the numerical distance between the two fractions and computing the coefficient of determination (r^2) for that relationship. The higher the value for r^2, the larger the numerical distance effect. Specifically, it was predicted that individuals who reported high levels of math anxiety or high amounts of calculator use would exhibit a smaller numerical distance effect, compared to those individuals who reported low levels of math anxiety or low amounts of calculator use. Also, it was predicted that individuals with higher arithmetic fluency would exhibit a larger numerical distance effect than those individuals with lower arithmetic fluency.

Method

Participants

Twenty-eight undergraduate students (21 female) from Texas A&M University – Commerce participated in the current study. The participants were volunteers from several freshman-level mathematics courses who took part in the study for partial course credit. The mean age was 27.3 years (range 18-55 years; median 25 years, standard deviation 8.41 years.

Experimental Stimuli and Measures

The set of fraction stimuli was set of 48 reduced, proper fraction pairs that consisted of three sets of 16 fractions. Each of the sets of 16 contained one of three critical fractions for comparison: 1/2, 1/3, or 2/3. In each group of 16, half of the fractions were less than the given critical comparison fraction, with the other half greater. Also, the left-side/right-side status of the larger fraction was equally distributed within each group of 16. No fraction pairs were repeated, but each individual fraction was presented twice (in different left/right positions) in comparison with two different critical fractions.

In addition, each participant completed a demographic survey asking for subjective ratings (on an integer scale of 1=low to 5=high) of their level of mathematics anxiety and tendency to use a calculator for routine computations. Each participant was also assessed on their arithmetic fluency by completing the Addition test and the Subtraction-Multiplication test from the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Dermen, 1976). The Addition test was composed of two pages of three-addend addition problems (for a total of 120 problems). The Subtraction-Multiplication test consisted of two pages of two-digit subtraction problems and two-by-one digit multiplication problems (for a total of 120 problems). Participants were allowed 2 minutes per page to correctly answer as many problems as they could. Arithmetic fluency was defined as the total number of correct answers on both tests.

Procedure

Participants were first given an instruction phase that consisted of three simple fraction comparisons: 1/2 vs. 1/3, 1/2 vs. 2/3, and 1/3 vs. 2/3. Participants were told to answer as quickly and accurately as possible. Feedback was presented in the form of an audible beep (for correct answers) and an audible buzz (for incorrect answers). Once the instruction phase was complete, participants were given a chance to ask any questions of the experimenter before the testing phase began.

During the testing phase, no feedback was given. Each trial began with the sentence, “Say ‘Go’ when ready,” presented in the center of the screen. Through a lapel microphone, the participant’s vocalization triggered the software to present a fraction pair, which remained on the screen until a button was pressed, the side indicating which fraction was larger in magnitude, or 15 seconds elapsed.

Results

A total of 1344 trials were administered. Of these trials, 30 trials were discarded due to either a failure in the experimental apparatus or a failure to respond within 15 seconds. Of the remaining 1314 trials, 177 were answered incorrectly, resulting in an overall error rate of 13.5%.

The median solution time across these remaining trials (including trials on which an error was committed) was 3142 ms, SD = 3311 ms.

At the item level, a regression analysis using the distance between the numerical values of fraction pairs as a predictor of median reaction time across all participants and trials showed a significant numerical distance effect, with numerical distance accounting for 44% of the variance in median reaction times ($t(46) = -6.06$, $p<0.001$). Remarkably, a regression analysis using the natural logarithm of the numerical distance between fractions as a predictor of median reaction time exhibited a numerical distance effect of almost equal size, with the logarithm of numerical distance accounting for 43% of the variance in median reaction times ($t(46)=-5.96$, $p<0.001$). Figure 1 shows side-by-side scatter plots representing both models. Since the logarithmic model accounted for no more variance in median reaction time than the linear model, no further consideration of the logarithmic model was made.

At the participant level, linear regression analyses predicting RT as a function of numerical distance showed that 18 of the 28 participants exhibited a significant numerical distance effect. For those participants with a significant numerical distance effect, numerical distance between fractions accounted for an average of 23.0% of the variance in reaction times (standard deviation = 8.3%). For participants who did not exhibit a significant numerical distance effect, numerical distance between fractions only accounted for 2.4% of the variance in reaction times (standard deviation 2.7%).

Further analysis of the contribution of individual differences to the size of the numerical distance effect showed marked differences (see Figure 2). To analyze the effect of mathematics anxiety, participants were grouped according to their subjective rating (1=low to 5=high) on the mathematics anxiety question in the demographic survey. Those participants rating themselves with a 1 or a 2 were classified as having “Low” mathematics anxiety, and those participants rating themselves as 4 or 5 were classified as having “High” mathematics anxiety. Four

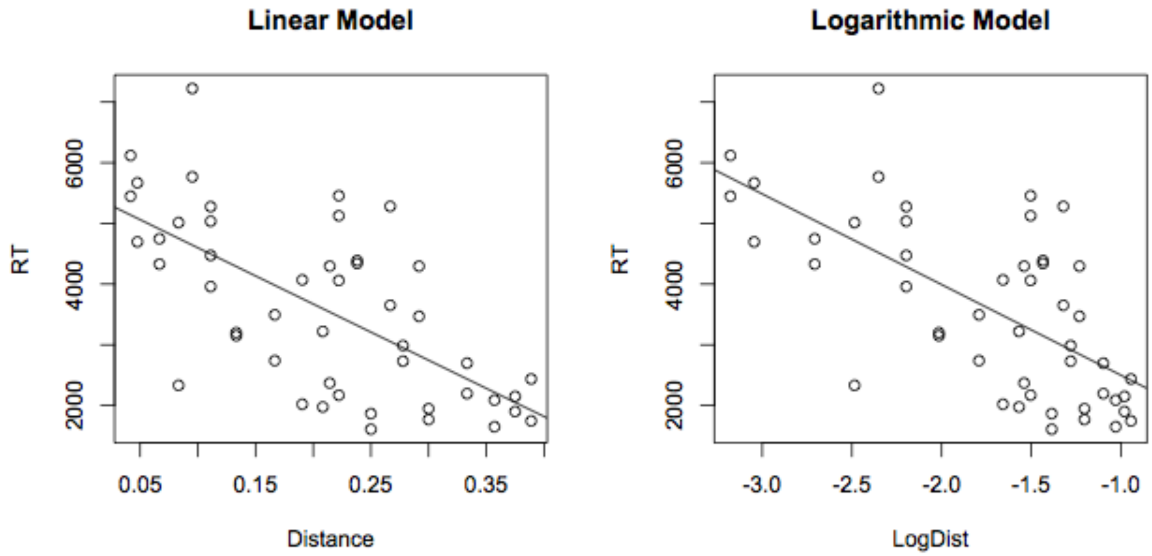

Figure 1. Scatter plots of median reaction time versus numerical distance between fractions. Both the linear and logarithmic models predict an equal amount of variance in the median reaction times.

participants rated themselves as 3, and were excluded from this analysis. Participants classified as Low Mathematics Anxiety exhibited a much greater numerical distance effect than those participants classified as High Mathematics Anxiety ($F(1,22) = 10.41$, $p=0.004$). That is, for participants with Low Mathematics Anxiety, numerical distance between fractions accounted for 23.1% of the variance in reaction time, whereas for participants with High Mathematics Anxiety, numerical distance only explained 9.2% of the variance in reaction times.

A similar analysis was conducted for daily calculator use. Participants were classified in a manner identical to the method for math anxiety. For reasons as above, five participants were excluded from this analysis. Participants classified as Low Calculator Use exhibited a much greater numerical distance effect than those classified as High Calculator Use. Numerical distance accounted for 26.2% of the variance in reaction time for those participants who rarely used calculators, compared to 9.9% for those who used calculators often.

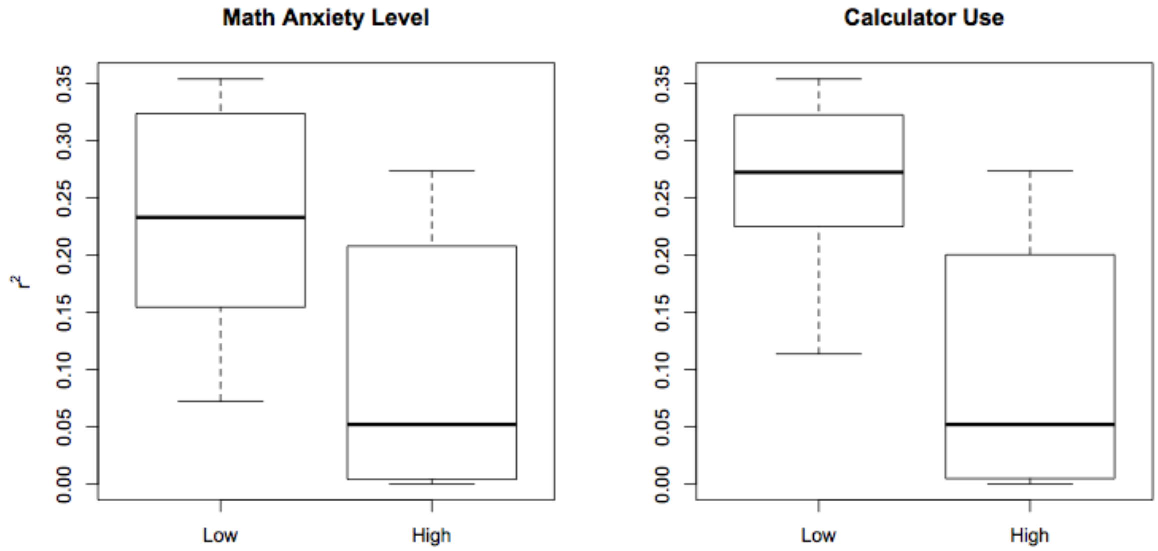

Figure 2. The size of the numerical distance effect as a function of individual differences in math anxiety level and daily calculator use.

Finally, the contribution of arithmetic fluency was analyzed by regressing an individual's coefficient of determination (r^2) with the score on the arithmetic fluency test as a predictor. This analysis showed virtually no effect of arithmetic fluency on the size of the numerical distance effect ($r^2 = 0.003, F(1,26)=0.08$).

Discussion

Participants tended to correctly select the larger of two presented fractions more quickly when the fractions presented were farther apart on the number line, compared to when the fractions were close together. This is typically thought to correspond to an integrated, magnitude-based representation that is akin to a mental number-line. This finding replicates the core finding of several recent studies (Faulkenberry & Pierce, 2010; Schneider & Siegler, 2010; Meert, Grégoire, & Noël, 2009) with one exception. The current study found that median RT is predicted best by a linear function of the numerical distance between fractions, whereas Schneider & Siegler (2010) showed that median RT was best predicted by the logarithm of the numerical distance. Nonetheless, the current data lends further support for the numerical distance effect as a robust effect in mathematical cognition.

The current study takes an additional step of considering individual differences as a predictor of the extent to which participants possess and use a well-developed mental number line for fractions. This extent was measured by the size of the numerical distance effect for each participant. Participants with a low level of mathematics anxiety tended to exhibit much larger numerical distance effects than those with a high level of mathematics anxiety. That is, individuals with low math anxiety tend to have more robust magnitude-based representations of fractions than their high math anxiety counterparts. Similarly, participants who use calculators very little in daily life were also found to have more robust magnitude-based representations of fraction than their counterparts who use calculators often. Perhaps surprisingly, arithmetic fluency had virtually no effect on a person's tendency to use a magnitude-based representation.

The use of regression parameters to quantify aspects of an individual's mental representation of number is not new (e.g., Salthouse & Coon, 1994; Geary, Frensch, and Wiley, 1993) and can illuminate many individual differences that would not be visible with raw reaction time data. Indeed, the individual regression parameters provide a way to standardize reaction time data that removes the influence of performance differences (such as prior knowledge and practice effects) and instead relies on within-subject patterns of performance.

In summary, the current study found that individuals with a high level of mathematics anxiety or a high propensity for calculator use tend to rely less on magnitude-based mental representations of fractions. Future research should attempt to study the consequences of these representational shifts, especially with respect to individuals' procedural and conceptual knowledge of fractions.

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