THE PRIMACY OF FRACTION COMPONENTS IN ADULTS' NUMERICAL JUDGMENTS

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A fundamental question in numerical cognition concerns how people make judgments about the magnitude of fractions. There is much debate around the issue of whether fraction representations are holistic or component-based. In the present study, we measured hand movements as people mentally compared fractions to ½. We found that participants' hands tended to move according to the size of components rather than the overall magnitude of the fraction. This indicates that people form an initial automatic representation that is tied to surface format (i.e., component-based), but later refine this representation according to task demands.

When skilled adults think about fractions, what do their representations look like? For instance, suppose you were asked to compare the fraction 3/7 to ½. Which is bigger? How do you make the decision? While there are multiple representations that can be deliberately formed depending on context (Lamon, 2005), we are interested in the automatic, unconscious mental representations that are formed when comparing fractions. The issue is by no means trivial: many recent studies have yielded equivocal results with respect to this issue. On one hand, some researchers believe that mental representations of fractions are based on the components of the fraction instead of the fractions numerical value, or magnitude (Bonato, Fabbri, Umilta, & Zorzi, 2007). On the other hand, others have found that people tend to immediately process the magnitude of fractions rather than the components (Meert, Gregoire, & Noel, 2009; 2010; Schneider & Siegler, 2010). Recent evidence indicates that the true answer may lie somewhere in the middle: Faulkenberry and Pierce (2011) concluded that the type of representation formed in a fraction task depends heavily on the strategy used.

The fundamental question becomes the following: in the 1-2 seconds that it takes an adult to compare two presented fractions, what types of mental representations does he/she form? With the existing equivocal evidence, it becomes difficult to make solid predictions. However, it may be possible to bridge these two seemingly disparate findings in the literature. Cohen Kadosh and Walsh (2009) have recently hypothesized a dual-process model of numerical representations. In this model, there is an initial, automatic representation that is directly tied to the surface format of the presented number. Later, there is a refinement of the automatic representation that is influenced by intentionality, resources, task demands, etc. It may be the case that the initial, automatic representation formed is directly tied to the components of the fraction, whereas the more refined representation uses magnitude information. We directly test this hypothesis in the present study. Critically, we use a hand-tracking paradigm (Spivey, Grosjean, & Knoblich, 2005; Freeman & Ambady, 2010) to gain insight into the online formation of fraction representations.

In the present study, we asked participants to quickly decide whether a presented fraction was smaller or larger than ½. During the task, we collected the streaming (x,y) coordinates of a computer mouse as they clicked on the correct response. By directly manipulating fraction magnitude and component size, we tested the selective influence of both factors on the trajectories of participants' hands as they made their decisions; this allows an unprecedented window into the formation of their mental representations (Freeman, Dale, & Farmer, 2011). If participants are indeed forming immediate representations based on components alone, then component size should have more of an influence on average hand trajectories than fraction magnitude. If, on the other hand, participants' immediate representations are based on magnitude, then we should see little difference in the trajectories of fractions with large components and those of equivalent magnitude with small components.

Method

Participants

26 undergraduate students (14 female, mean age 23.1 years) participated in exchange for partial course credit.

Stimuli

The fractions presented to participants were chosen by crossing the factors of fraction magnitude (smaller than ½, larger than ½) and component size (larger than 5, smaller than 5). Within each of these four cells, we chose two fractions (see Table 1).

Procedure

Participants were told that for each trial, they would be asked to quickly and accurately choose whether the presented fraction was greater or smaller than the target fraction ½. At the

Table 1

Fraction Stimuli, presented as a function of Magnitude and Component Size

	Component size	
Fraction magnitude	Smaller than 5	Larger than 5
Smaller than 1/2	1/4, 1/3	2/8[a], 3/9[a]
Larger than 1/2	2/3, 3/4	6/9, 6/8

Note: [a] To preserve magnitude in this condition, only denominators are larger than 5.

beginning of each trial, a button labeled START appeared at the bottom center of the screen, along with the two response labels SMALLER and LARGER presented in the upper left and right corners of the screen. Participants completed one block of trials with the labels ordered SMALLER -- LARGER from left to right, and the other block had labels ordered LARGER – SMALLER from left to right. The order of these blocks was counterbalanced across participants.

After participants clicked the start button, one of the 8 stimulus fractions randomly appeared in the center of the screen. Participants were then required to quickly click on the response label appropriately designating whether the presented fraction was larger or smaller than ½. During these responses, we recorded the streaming (*x,y*)-coordinates of the participants' computer mouse movements (with a sampling rate of approximately 70 Hz). To present stimuli and record mouse trajectories during responses, we used the MouseTracker software package (Freeman & Ambady, 2010). In order to guarantee that mouse trajectories reflected online processing, we instructed participants to begin moving their computer mouse as quickly as possible. In the event that the mouse initiation time exceeded 250 ms, a message appeared on the screen after the participant's response, instructing them to start moving earlier on future trials, even if they were not completely sure of their response. In total, each participant completed 120 trials (60 in each response label ordering).

Results and Discussion

To prepare the raw mouse trajectory data for analysis, we performed an initial preprocessing with the MouseTracker software package (Freeman & Ambady, 2010). All mouse trajectories were rescaled into a standard coordinate space (*x*-coordinate range: -1 to 1;

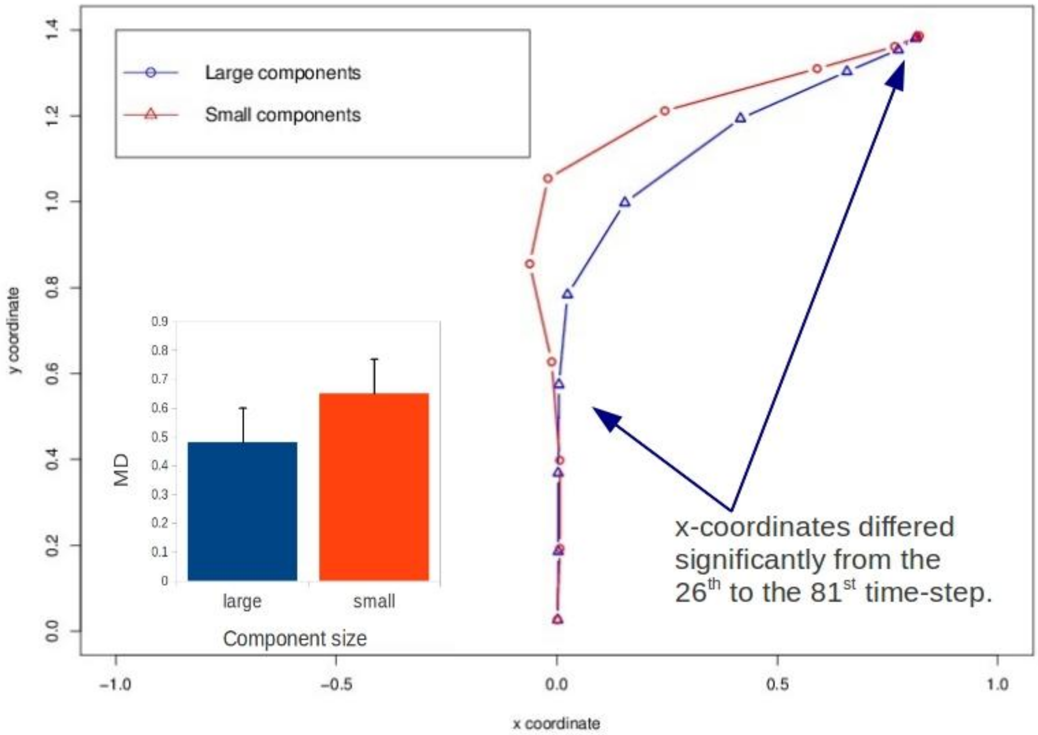

Figure 1: Mean trajectories and MD values for large fractions as a function of component size.

y-coordinate range: 0 to 1.5). In addition, to remove the confound of varying response times, all raw trajectories were normalized (via linear interpolation) to consist of 101 time steps. This step was critical in order to allow us to average across trials with differing time durations. As an index of trajectory complexity, we measured the degree to which the incorrect response alternative influenced participants' decisions by computing the maximum deviation (MD): the largest perpendicular deviation between the actual trajectory and the ideal response trajectory, represented by a straight line from the trajectory's starting point and the correct response (see Figure 1). Subsequent analyses were performed using linear mixed effects modeling (Pinheiro & Bates, 2000; Bates, Maechler, & Bolker, 2011) with the R statistical package (R Development Core Team, 2011).

Fraction performance

Participants were very quick and accurate to judge whether presented fractions were smaller or larger than 1/2. Across 3,120 trials, only 133 were in error (4.3% error rate). Overall, the mean reaction time of the correct trials was 1247 ms (*SD* = 546 ms). Outlier screening was initially performed; we rejected correct trials if the respective RT exceeded 3 SD away from the mean. 58 trials were discarded (1.9%). To analyze the influence of component size on the decision process when participants made their responses, we separately considered those fractions that were larger than ½ and those that were smaller than ½.

Large fractions

All fractions analyzed herein were larger than 1/2; hence, the correct response for all stimuli was LARGER. For ease of visualization and interpretation of these mouse trajectories, we remapped all trajectories to the right side of the display. Then we computed a mean trajectory for fractions with small components (2/3, 3/4) and fractions with large components (5/6, 7/8). As can be seen in Figure 1, trajectories for fractions with small components exhibit a great deal of continuous attraction toward the incorrect alternative (SMALLER), compared with fractions having large components. This effect was statistically significant; as indexed by maximum deviation (MD), trajectories for fractions with small components (fitted MD = 0.50) were significantly attracted toward the answer SMALLER relative to fractions with large components (fitted MD = 0.18), $t = 12.28$, $p < 0.0001$.

Small fractions

Similar to the previous analysis, all fractions analyzed herein were smaller than ½; hence, the correct response for all stimuli was SMALLER. This time, we remapped all trajectories to the left side of the display. Then we computed a mean trajectory for fractions with small components (1/3, 1/4) and fractions with large components (1/6, 1/8). As indicated in Figure 2, there was a large difference between the trajectories for fractions with small components and fractions with large components. Again, this effect was statistically significant; as indexed by maximum deviation (MD), trajectories for fractions with large components (fitted MD = 0.45) were significantly attracted toward the answer SMALLER relative to fractions with large components (fitted MD = 0.19), $t = 18.04$, $p < 0.0001$.

The role of magnitude?

Previous studies (e.g., Meert, Gregoire, & Noel, 2009; Faulkenberry & Pierce, 2011) have shown that people tend to process the overall magnitude of fractions during fraction comparison tasks. Typically, this effect is quantified by regressing reaction times with the distance between fractions to be compared. The presence of a *negative slope*, known as the numerical distance

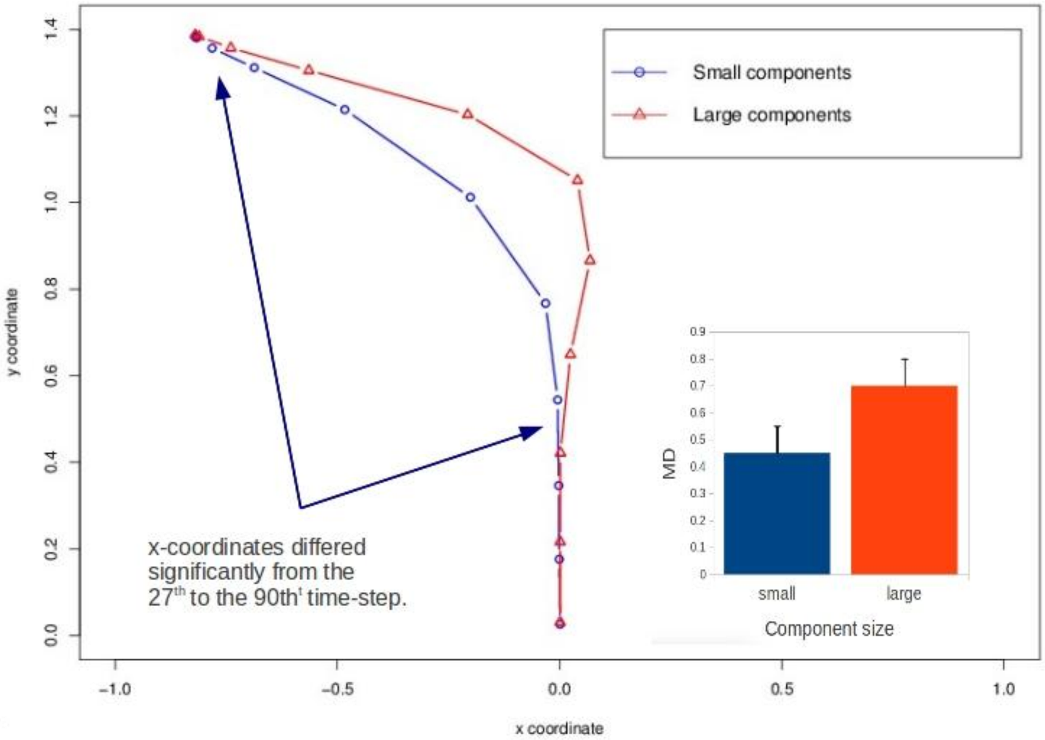

Figure 2: Mean trajectories and MD values for small fractions as a function of component size.

effect (NDE; Moyer & Landauer, 1967), is typically taken as evidence of participants' magnitude-based representations of numbers.

To assess whether participants in the present study attended to the overall magnitude of the presented fractions, we computed a linear mixed-effects model (Pinheiro & Bates, 2000) in R using the `lmer` package (Bates, Maechler, & Bolker, 2011). At the first stage, we computed a mixed-effects model with RT as a dependent measure, distance as a fixed effect, and participant as a random effect. The presence of the random effect term allows the intercept to vary for each participant while assessing a fixed slope, or effect, for distance across all participants. Critically, this model was fitted with a slope estimate for the distance fixed effect of -374.8 ($t = 3.27$). As this modeling is done within a Bayesian framework, "significance" is assessed via other means. One method is to compute a Bayesian analogue of a confidence interval for the slope (the 95% HPD, or highest posterior density) using 10,000 bootstrap samples. We found the 95% HPD to be (-597.4, -141.9). Both of these pieces of evidence indicate that numerical distance is indeed a significant predictor of reaction times. In other words, participants seem to be attending to the numerical value of the presented fractions, even though their mouse trajectories indicate that their decisions are quite influenced by the size of the components in the fractions.

The present data supports the dual-process model of Cohen Kadosh and Walsh (2009), whereby participants' initial, automatic representation that is directly tied to the surface format of the presented fraction. This was evidenced by the consistent effect that component size had on participants hand trajectories in the fraction comparison task: when component size was inconsistent with the overall magnitude of the fraction (i.e., large components, but small overall magnitude), participants hands tended to drift away toward the incorrect answer before eventually settling in picking the correct one. Also predicted by the dual process model is a later refinement of the automatic representation that is influenced by intentionality, resources, task demands, etc. In the present experiment, we hypothesized that this would be where the magnitude representation would come into play. Indeed, the current data supports this; through mixed effects modeling, we were able to find a consistent negative slope when regressing reaction times on distance, indicating that fractions farther from ½ took less time to respond to than did fractions that were close to ½. This is a classic marker of magnitude-based representations (Moyer & Landauer, 1967).

General Discussion

The present research may provide a bridge between some seemingly contradictory findings in recent research on fraction representations. We found that adults form fraction representations that attend to both the components *and* the magnitude of a fraction. While this may seem obvious, the magnitude part of these results is a bit trickier to resolve. In the present task (deciding if a fraction is greater than or less than ½), there is no reason, *a priori,* for someone to think about "how big" the fraction is. Indeed, the task could easily be taught to a child using without having to have a solid knowledge of fractions. However, the present data indicates that magnitude does indeed play a part in our mental processing of fractions. This has important ramifications for teaching: since magnitude is a critical part of successful adult representations of fractions, it is important that children gain a knowledge of fractions not only from a symbolic, component-driven view, but also their underlying numerical values.

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