HOW THE HAND MIRRORS THE MIND: THE EMBODIMENT OF NUMERICAL COGNITION

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Much knowledge about how the mind does mathematics is based on the traditional, computer-based metaphor of cognition that assumes cognition is stage-based and independent of the motor cortex. In the present study, I provide evidence for an alternative view. I recorded participants' hand movements as they chose the correct parity (odd/even) for single-digit numerals. Distributional analyses of these movements indicated that responses resulted from competition between parallel and partially-active mental representations rather than occurring in discrete stages. Furthermore, this competition was carried through to the motor cortex, indicating that numerical representations are more tied to bodily affordances than previously thought.

Researchers have been investigating mathematics learning for many years, particularly through the paradigm of cognitive psychology. From early attempts to understand how arithmetic facts are organized (Ashcraft & Battaglia, 1978) to formal models that specify the various cognitive processes involved in mathematical problem solving (Anderson, 2005), most of these studies have made the implicit (albeit, metaphorical) assumption that the mind operates like a computer. That is, perception informs cognition, and cognition informs action. In this view, higher-level cognitive systems (memory, executive control, etc.) are thought to be quite separate from the lower-level systems (perception, motor action).

This modular, discrete-systems approach to cognition has been the fundamental underpinning of most of our understanding of how the mind does mathematics. From the point of view of "mathematics is a collection of abstract ideas," it makes sense that mathematical objects could be learned and operated on in a purely abstract fashion without any interaction with other (non-cognitive) neural systems, such as the motor cortex. However, Lakoff and Nunez (2000) proposed the hypothesis that mathematics is learned through conceptual metaphor, a mechanism for converting embodied (sensori-motor) reasoning to abstract reasoning. At the time, unfortunately, their view was almost entirely philosophical, and it elicited much debate between cognitive scientists and mathematicians. Without behavioral evidence, the debate would be sure to stay within the realm of philosophy, and as such, not be widely accepted among mathematicians and psychologists alike.

In recent years, however, other cognitive scientists have proposed a view similar to that of

Lakoff and Nunez: specifically, that the human mind is not a modular computer, but rather a rich, dynamic system of parallel and partially-active representations (Spivey, 2007). In this view, decisions are not made through modular "switches," but instead are the result of competition among many different partially-activated responses, simultaneously informed by feedback from many other systems, including (especially) the motor systems.

The canonical example of this view is found in the language-processing literature (Spivey, Grosjean, & Knoblich, 2005). In a language-comprehension task, they asked participants to listen to words and, with a computer mouse, choose the picture that correctly represented the spoken word. During this task, they measured participants' hand positions by continuously recording the (x,y) coordinates of their mouse. They found that when words were phonetically similar (CANDY versus CANDLE, see Figure 1), the mouse tracks tended to deviate toward the incorrect alternative early in the response process, but eventually settle in to the correct answer. This is commonly taken as evidence for an embodied view of cognition, where responses result from a dynamic competition between partially-active, unstable mental representations. In the classic, modular view of cognition, the hand positions would not be so sensitive to influence from the decision process, as the motor system would not be called upon until the decision was made in the language center of the brain.

Until now, no studies have investigated the processing of numerical information within such a continuous, embodied-cognition framework. This is unfortunate, as the work of Lakoff and Nunez (2000) has set the stage for such research to tease apart the contributions of different cognitive and perceptual systems to numerical cognition. In the present study, I used the hand-tracking paradigm of Spivey (2007) to capture the formation of numerical representations during a parity judgment task. Participants quickly judged whether single digit numbers were even or odd. Responses were either consistent with spatial orientation of numbers (i.e., small numbers on left side or large numbers on right side) or inconsistent (i.e., small numbers on right side, large numbers on left side. Two competing predictions were then tested. If numerical cognition is indeed part of an embodied system, then hand trajectories in the *inconsistent* condition should show a pull in the direction of the incorrect alternative, reflecting a settling of partial activations of response alternatives during the response. If, on the other hand, numerical cognition is

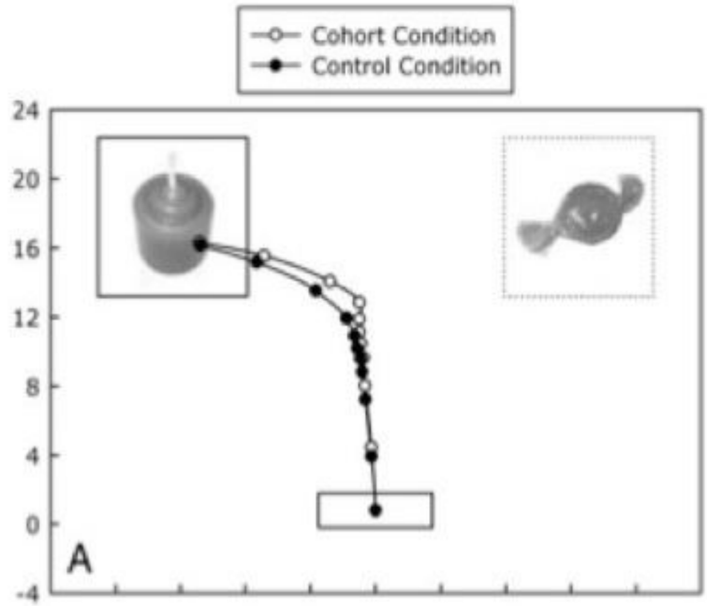

Figure 4: Words that are phonological similar (candy versus candle) show a competition throughout the response.

modular and stage-based, then we should see little attraction toward the incorrect alternatives, since the incompatibility would be resolved before the motor output stage.

Method

Participants

45 undergraduate students (35 female, mean age 24.3 years) participated in the present study.

Stimuli and Procedure

Single digit numerals (excluding 5, as is common in the numerical processing literature) were presented on a computer screen using the software package MouseTracker (Freeman & Ambady, 2010). Participants were told that, on every trial, a number would appear in the center of the screen, and they would be asked to choose, as quickly as possible, whether the number was even or odd. After participants clicked a "Start" button centered at the bottom of the screen, response labels "Even" and "Odd" appeared at the top left and right of the screen (the order of these labels was switched once midway through the experiment). Participants then clicked on the correct of these two options; while doing this, I recorded the streaming (x,y) coordinates of the computer mouse approximately 70 times per second. Each participant completed 640 trials. This yielded a rich data set of hand trajectories, which in the spirit of Spivey and colleagues (2005) directly reflects the mental processes that occurred during the numerical decision making process.

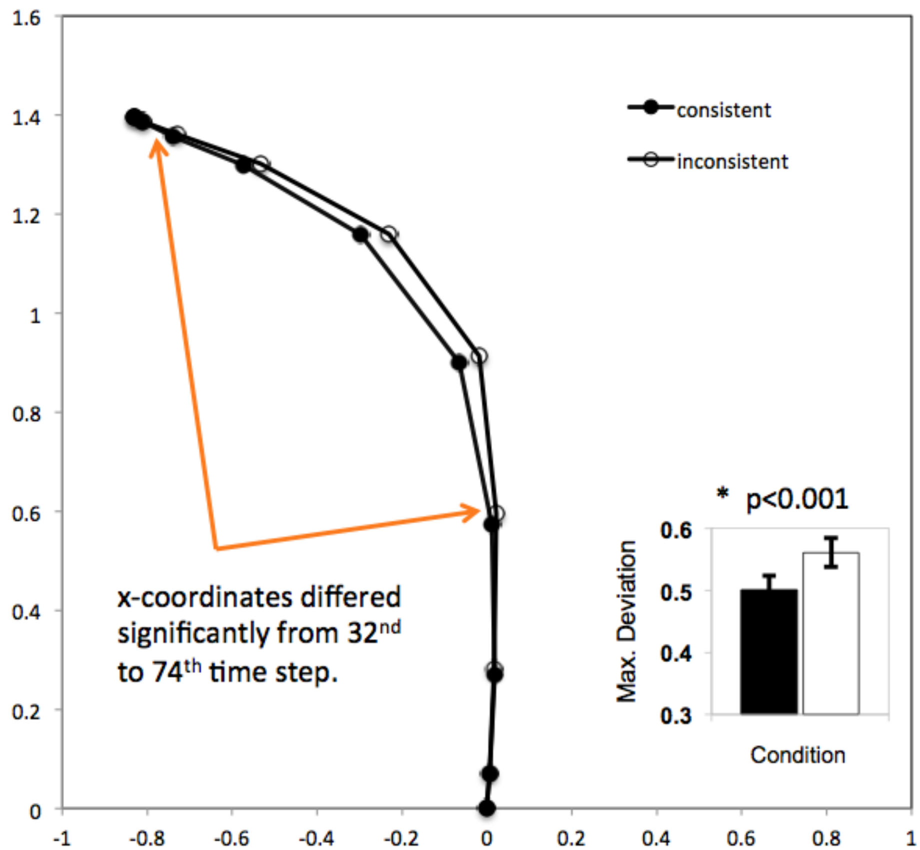

Figure 5: Average hand trajectories during the numerical parity task, separated as a function of spatial compatibility.

Results and Discussion

All hand trajectories were rescaled into a standard coordinate space, [-1,1] x [0,1.5]. To analyze movements independent of reaction times, I used linear interpolation to normalize all trajectories to consist of 101 times steps. This is important so that trajectories of differing time scales can be averaged over multiple trials. In addition, for ease of visualization, all trajectories for responses on the right-hand side of the screen were reflected to the left side of the screen.

The first analysis is with respect to the hand trajectories in each of two spatial compatibility conditions. On *consistent* trials, participants responded to small numbers (1,2,3,4) on the left side of the screen and large numbers (5,6,7,8) on the right side of the screen. On *inconsistent* trials, these were reversed. These conditions are motivated by the robust finding that most English-speaking adults have an implicit left-right number orientation (Dehaene, Bossini, & Giroux, 1993).

To analyze the hand trajectories, I computed an average trajectory across all participants for each of the two spatial compatibility conditions. As can be seen in Figure 2, hand trajectories in the inconsistent condition are a bit "pulled away" from the trajectories for the consistent condition. One interpretation of this is that the trajectories in the inconsistent condition continuously attracted toward the incorrect response alternative throughout much of the response, indicating a high degree of competition between the two response alternatives. Indeed, across all trials, the average trajectory was significantly closer in proximity to the incorrect response alternative from the 32nd to the 74th time step.

For a trial-by-trial index of the degree to which the incorrect response alternative was partially active, I computed the *maximum deviation:* the largest positive x-coordinate deviation from an ideal response trajectory (a straight line between the start button and the response) for each of the 101 time steps. As indexed by maximum deviation, trajectories for inconsistent responses ($M=0.56$, $SE=0.02$) were significantly more attracted to the incorrect response alternative, compared with trajectories for consistent responses ($M=0.50$, $SE=0.02$), $t(44)=6.41$, $p < 0.0001$.

Across both measures, the data reflect that during the numerical decision process, participants formed partially-active representations of both response alternatives until the "winning" representation was stabilized and the correct answer was chosen. Initially, this seems to support the embodied view of cognition. However, an alternative explanation could instead explain the data. It could be the case that the smooth, continuous attraction we are seeing is the result of averaging across trials. For example, if some trials showed zero attraction (i.e., the participants' hands moved directly toward the correct answer) and other trials were sharply deflected midflight after realization of an error, the appearance of the average trajectories would be smooth even though the cognitive processes involved were modular (that is, motor responses were not initiated until the decision was made). In this case, if we were to look at the distribution of the maximum deviation values, it would be distinctly bimodal; simply put, some of the values would be small (indicating direct trajectories) and others would be large (reflecting the midflight correction of an almost incorrect response).

To test whether this is the case, I performed a distributional analysis on the collection of maximum deviation values across all trials. Each of the 28,800 values (640 values for each of 45 participants) was converted to a z-score. Figure 3 depicts the distribution of these maximum

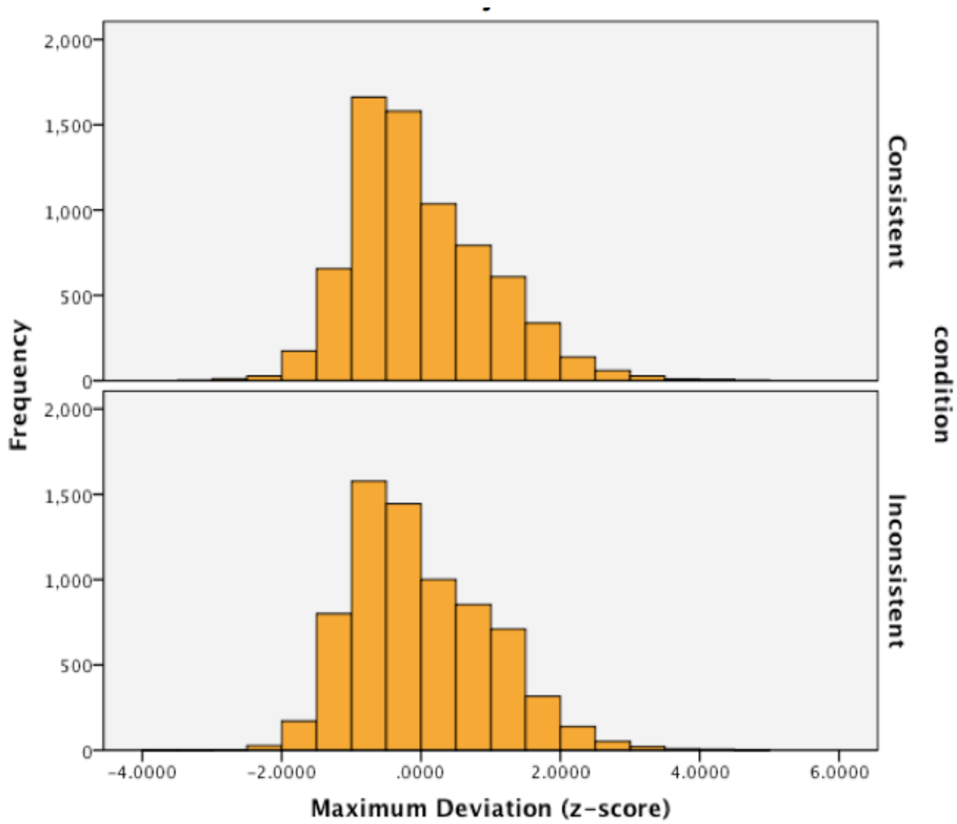

Figure 6: Distributions of maximum deviation values for consistent and inconsistent trials

deviation values for both consistent and inconsistent trials. Notice, critically, that the inconsistent trials do not differ in shape from the consistent trials, nor do they appear bimodal. Modality analysis confirms that the distribution of inconsistent trials is indeed not bimodal: the computed coefficient of bimodality was 0.423, with $b > 0.555$ representing the minimum value for a distribution to be considered bimodal. Also, a Kolmogorov-Smirnov test confirms that the distribution of values on consistent trials is not significantly different from those in inconsistent trials ($z = 1.33$, $p > 0.06$). These data indicate that the distribution of maximum deviation values is not bimodal, and that the smooth, continuous attraction away from the correct answer in the inconsistent trials is *not* the result of participants' quickly correcting their fast, incorrect initial responses.

In summary, we found an interesting pattern of responses when people are making quick judgments about the parity of a number (whether it is even or odd). Particularly, the size of the number affects the dynamics of our hand responses (even though the size is irrelevant to the task). This effect was captured by looking at the streaming path of computer mouse coordinates as participants selected the correct response label (even or odd). There seemed to be an automatic activation of numerical size that was carried out in participants hand movements (see Figure 2). This directly supports the hypothesis of embodied cognition. However, this pattern could have also resulted from an “averaging” across trials; on inconsistent trials, the mouse movement could have initially been in the wrong direction, then immediately corrected midflight to the correct response alternative. However, an trial-by-trial analysis rules this possibility out (see Figure 3).

Together, these results comprise an important first step in establishing the embodiment of numerical cognition. From the work of Lakoff and Nunez (2000), an important philosophical claim was made: mathematics is entirely the creation of humans using entirely human qualities. In other words, mathematics as we know it could not have been “invented” without the bodily affordances that make us human. While this claim may seem more the realm of philosophers and science fiction writers, the present results provide some evidence that even numerical decisions are intimately tied to our bodily states.

General Discussion

The results of the present study indicate that numerical processing does not take place independently from our bodily states. Specifically, we found that when tracking hand movements in even the most simple task (a parity judgment task), the hand movements reflected a continuous, dynamic system of partially activated cognitive states that would not be possible in the traditional, computer-based metaphor of mind.

At first, it may be difficult to see how these results relate to discussions in mathematics education. Indeed, the results from such research are valuable to mathematics educators because, together, they lend theoretical support to the idea of embodied mathematics. Embodied mathematics is the view that mathematics is not completely an abstract, or “pure,” discipline, but rather is the product of a conceptual system that is ultimately grounded in bodily states. In a sense, this “humanizes” mathematics. This kind of evidence also tells us that since mathematics is tied to our body-grounded conceptual systems, it should be taught from that point of view. That is, as Nunez and colleagues put it:

> “Students (and teachers) should know that mathematical theorems, proofs, and objects are about ideas, and that these ideas are situated and meaningful because they are grounded in our bodily experience as social animals.” (Nunez, Edwards, & Matos, 1999, p. 62).

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