Hand Movements Reflect Competitive Processing in Numerical Cognition

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Abstract

Traditional models of numerical cognition are based on the computer-based metaphor of cognition that assumes numerical judgements are stage-based and independent of bodily effectors. However, recent studies have indicated that the traditional metaphor may be inadequate for describing the processes involved in numerical decisions. In the present study, I provide further evidence that number processing proceeds in a continuous, competitive manner tightly coupled with feedback from the motor system. 45 adult participants' hand movements were recorded as they used a computer mouse to choose the correct parity (odd/even) for single-digit numerals. Distributional analyses of these hand movements indicated that responses resulted from competition between parallel and partially-active mental representations rather than occurring in discrete stages.

Keywords: Numerical judgements, parity task, hand tracking, response distribution

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Researchers have investigated mathematics learning and performance for many years, particularly through the lens of cognitive psychology. From early attempts to understand how arithmetic facts are organized in long-term memory structures

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(Ashcraft & Battaglia, 1978) to formal models that specify the cognitive processes involved in mathematical problem solving (Anderson, 2005), most of these studies have made the implicit (albeit, metaphorical) assumption that the mind operates like a computer (Fodor, 1983). That is, perception informs cognition, and cognition informs action. In this view, higher-level cognitive systems (memory, executive control, etc.) are thought to be quite separate from the lower-level systems such as sensation/perception and those responsible for motor affordances.

This modular, discrete-systems approach to cognition has been the fundamental underpinning of most of our understanding of how the mind does mathematics. If one starts with the generally accepted axiom that mathematics is a collection of abstract ideas, then it should make sense that mathematical objects could be learned and operated on in a purely abstract fashion without any interaction with other (non-cognitive) neural subsystems, such as the motor cortex. However, Lakoff and Núñez (2000) proposed that mathematics is learned through conceptual metaphor, a mechanism for converting embodied, sensorimotor reasoning to abstract reasoning. At the time, much of their argument was almost entirely philosophical, and it elicited much debate between cognitive scientists and mathematicians. Without converging behavioral evidence, the debate would be sure to stay within the realm of philosophy, and as such, not be widely accepted among mathematicians and psychologists alike.

In recent years, cognitive scientists working in other areas have proposed a similar view that the human mind is not a modular computer, but rather a rich, dynamic system of competing parallel and partially-active representations (Spivey, 2007; Freeman, Dale, & Farmer, 2011). In this "cognition-as-competition" view, decisions are not made through modular processes akin to "switches," but instead are the result of competition among many different partially-activated responses, simultaneously informed by feedback from many other systems, including the motor systems. This framework highly related to embodied cognition, where the bodily affordances such as sensors and effectors actually contribute to the processes involved in cognition, rather than simply serving as end points in a linear processing chain (see Barsalou, 1999; Wilson, 2002).

A seminal example of the continuous, cognition-as-competition view can be found in the language-processing literature (Spivey, Grosjean, & Knoblich, 2005). In a language-comprehension task, Spivey et al. asked participants to listen to words and, with a computer mouse, choose the picture that correctly represented the spoken word. During this task, they measured participants' hand positions by continuously recording the (x, y)-coordinates of the computer mouse as it traveled on the screen while participants made their choice. They found that when words were phonetically similar (e.g., CANDY versus CANDLE), the mouse tracks tended to deviate toward the incorrect alternative early in the response process, but eventually settle in to the correct answer. This is commonly taken as evidence for an embodied view of cognition, where responses result from a dynamic competition between partially-active, unstable mental representations. Contrasted with the classic, modular view of cognition, the hand positions would not be so sensitive to influence from the decision process, as the motor system would not be called upon until the decision was made in the language center of the brain.

The idea that numerical processing may be tightly coupled to bodily affordances is not entirely new. One of the more robust results in numerical cognition research is the SNARC effect (Spatial-Numerical Association of Response Codes; Dehaene, Bossini, and Giraux, 1993). Participants elicit a SNARC effect in tasks that require a left-hand or right-hand response to a numerical stimulus; specifically, people tend to respond faster to small numbers with the left hand and to large numbers with the right hand (although, the specific mapping seems to be highly dependent on culture (e.g., Shaki, Fischer, and Petrusic, 2009). The effect is often taken as evidence for an implicit spatial arrangement of a mental number line.

Indeed, recent computational models of numerical decision processes (e.g., Gevers, Verguts, Reynvoet, Caessens, and Fias, 2006) have built in architecture that allows for continuous competition between competing response codes that are simultaneously activated by numerical magnitude and the task instructions. While the model does not make specific predictions about response trajectories that participants would make while completing numerical decision tasks, the underlying dual-route architecture is clearly in the spirit of the "cognition-as-competition" view. Along this line, Santens, Goossens, and Verguts (2011) explicitly measured response trajectories as participants completed a magnitude comparison task (deciding whether presented numbers were greater than or less than 5). They found that hand trajectories curved more greatly as a function of decreasing distance from 5, indicating that numbers that were closer to 5 in magnitude engendered more competition between numerical codes. In a similar experiment, Song and Nakayama (2008) found data that mirrored those of Santens et al. (2011). However, Song and Nakayama (2008) interpreted their results in terms of visuo-spatial representations of number along a mental number line. Whereas Santens et al. (2011) and Song and Nakayama (2008) reach the same conclusion with respect to visuo-spatial coding of numerical information (see also Gevers et al., 2010), only Santens et al. (2011) interpret their results in terms of competitive processing.

One limitation to the study of Santens et al. (2011) is that they did not address one possible alternative explanation for their data. It is possible that curved trajectories can result from averaging two types of extreme response trajectories (Freeman & Dale, 2012): one path that is directly to the correct answer, and another path that begins in the wrong direction (i.e., the participant is "tricked"), but is sharply corrected in midflight. This type of behavior would result in a bimodal distribution of response trajectories. To conclude that curved trajectories really stem from competitive processes, one must rule out this alternative.

In the present study, I used the hand-tracking paradigm of Spivey (2007) to capture the temporal dynamics of the formation of numerical representations during a parity judgement task. Participants quickly judged whether single digit numbers were even or odd. Responses were either consistent with spatial orientation of numbers (i.e., small numbers on left side or large numbers on right side) or inconsistent (i.e., small numbers on right side, large numbers on left side). Two competing predictions were then tested. If numerical decisions are reached via continuous, competitive processes, then hand trajectories in the inconsistent condition should show a pull in the direction of the incorrect alternative, reflecting a settling of partial activations of competing response alternatives during the response. If, on the other hand, numerical cognition is modular and stage-based, then we should see little attraction toward the incorrect alternatives, since the incompatibility would be resolved before the motor output stage.

Method

Participants

45 undergraduate psychology students (35 female, mean age 24.3 years) participated in the present study for partial course credit. All participants reported being right-hand dominant.

Stimuli and Procedure

Single digit numerals (excluding 5) were presented on a computer screen using the software package MouseTracker (Freeman & Ambady, 2010), freely available as a download from http://www.dartmouth.edu/~freemanlab/mousetracker/. Figure 1 depicts the sequence of stimuli in each experimental trial. Participants were told that on every trial a number would appear in the center of the screen, and they would be asked to choose, as quickly as possible, whether the number was even or odd. Each trial started with a blank screen presented for 1000 ms, followed by a screen that displayed the response labels EVEN and ODD at the top left and right of the screen, respectively. The order of these labels was switched once midway through the experiment; half of the participants started with the EVEN-ODD ordering, while the other half began with the ODD-EVEN ordering. After 1000 ms, a START button appeared. Once the START button was pressed, one of the stimulus numerals appeared in the center of the screen. Participants then clicked on the correct of these two options; while doing this, the software recorded the streaming (x, y) coordinates of the computer mouse approximately 70 times per second.

To ensure that mouse trajectories would reflect as much online processing as possible, participants were asked to begin moving their mouse as soon as possible. If initial mouse movement did not begin within 250 ms, a message appeared on the screen informing the participant to begin moving the mouse earlier. Each participant completed 640 trials, consisting of each of the 8 stimulus numerals presented 40 times over each response label ordering (2 blocks of 320 trials).

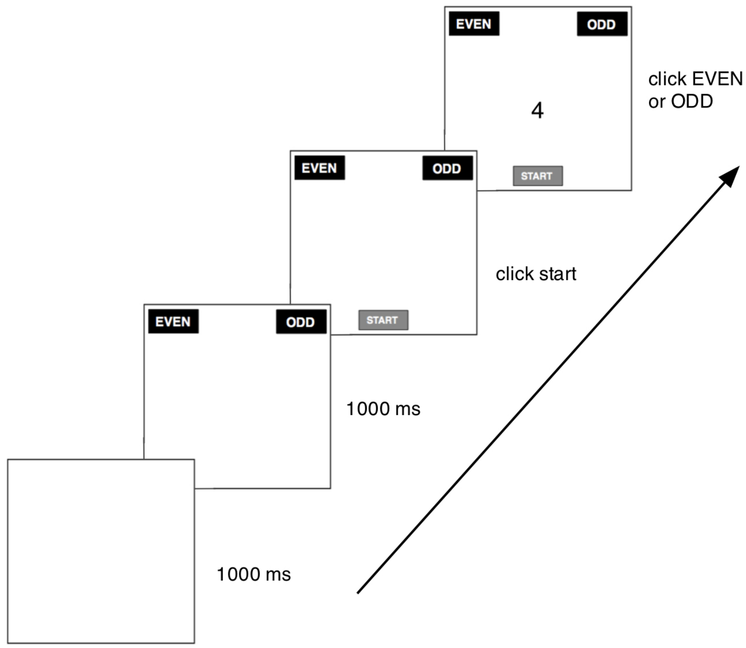

Figure 1. An example trial presentation. Participants were asked to click on the correct parity of the presented numeral. On half of the trials, the position of the response labels EVEN and ODD were reversed.

Results

Participants completed 28,800 parity judgement trials. Of these, there were only 259 errors (error rate = 0.9%). There were 117 errors in the consistent condition and 142 errors in the inconsistent condition. As indexed by a 2-sample proportion test, this difference in errors was not statistically significant, $z = 1.6$, $p > 0.11$. Error trials were subsequently discarded and further analyses were performed only on the remaining 28,541 correct trials.

Mean reaction times are presented in Table 1. Participants took 29 ms longer to complete inconsistent trials compared to consistent trials, $t(44) = -6.70$, $p < 0.001$. This difference was manifested as differences in actual movement duration as opposed

Table 1
Mean intial reaction times, movement durations, and total time (in ms) as a function of number-space consistency (SE in parentheses)

	Initial RT	Movement time	Total time
Consistent	80 (5)	953 (21)	1033 (21)
Inconsistent	81 (5)	981 (22)	1062 (21)

to differences in time for initiation of mouse movement (80 vs 81 ms, $t(44) = 0.80$, $p > 0.4$). While this difference reflects what would be predicted from the SNARC effect, it is the trajectories *during* the response process that are of primary interest in the present study.

As is common in hand tracking experiments (Freeman & Ambady, 2010), all hand trajectories were rescaled into a standard coordinate space, $[-1, 1] \times [0, 1.5]$. To analyze movements independent of reaction times, I used linear interpolation to normalize all trajectories to consist of 101 time steps. This is important so that trajectories of differing time scales can be averaged over multiple trials. In addition, for ease of visualization, all trajectories for responses on the right-hand side of the screen were reflected to the left side of the screen.

To analyze the hand trajectories, I computed an average trajectory across all participants for each of the two spatial compatibility conditions. As can be seen in Figure 2, hand trajectories in the inconsistent condition are a bit “drawn away” from the trajectories for the consistent condition. This is consistent with the prediction of Gevers et al. (2006) that inconsistent trials produce a conflict between magnitude representations (e.g., 2=small) and visuo-spatial representations (e.g., small=left), and resolution of this conflict manifests as a trajectory signature that appears as attraction toward the incorrect alternative. This trajectory pattern indicates a high degree of competition between the two response alternatives, as the average trajectory was significantly closer in proximity to the incorrect response alternative from the 32nd to the 74th time step.

For a trial-by-trial index of the degree to which the incorrect response alternative was partially active, I computed the maximum deviation (MD): the largest positive x-coordinate deviation from an ideal response trajectory (a straight line between the start button and the response) for each of the 101 time steps. MD values were then averaged for each of the 45 participants by consistency condition and subjected to a paired-samples t-test. As indexed by maximum deviation, trajectories for inconsistent responses (M=0.56, SD=0.15) were significantly more attracted to the incorrect response alternative, compared with trajectories for consistent responses (M=0.50, SD=0.15), $t(44) = 6.41$, $p < 0.0001$. To assess the evidence in favor of the hypothesis that MD values differ significantly as a function of consistency condition, I computed a JZS Bayes factor (Rouder, Speckman, Sun, Morey, & Iverson, 2009). The JZS Bayes factor for the present data was $B_{10}^{JZS} = 174,550$, indicating considerable evidence that, at least as indexed by maximum deviation, trajectories in the inconsistent condition are partially attracted toward the incorrect response alternative before settling in to the correct response.

Across both measures, the data reflect that during the numerical decision process, participants formed partially-active representations of both response alternatives until the winning representation was stabilized and the correct answer was chosen. Initially, this seems to support the hypothesized competition-driven view of numerical cognition (Santens et al., 2011; Gevers et al., 2006). However, an alternative explana-

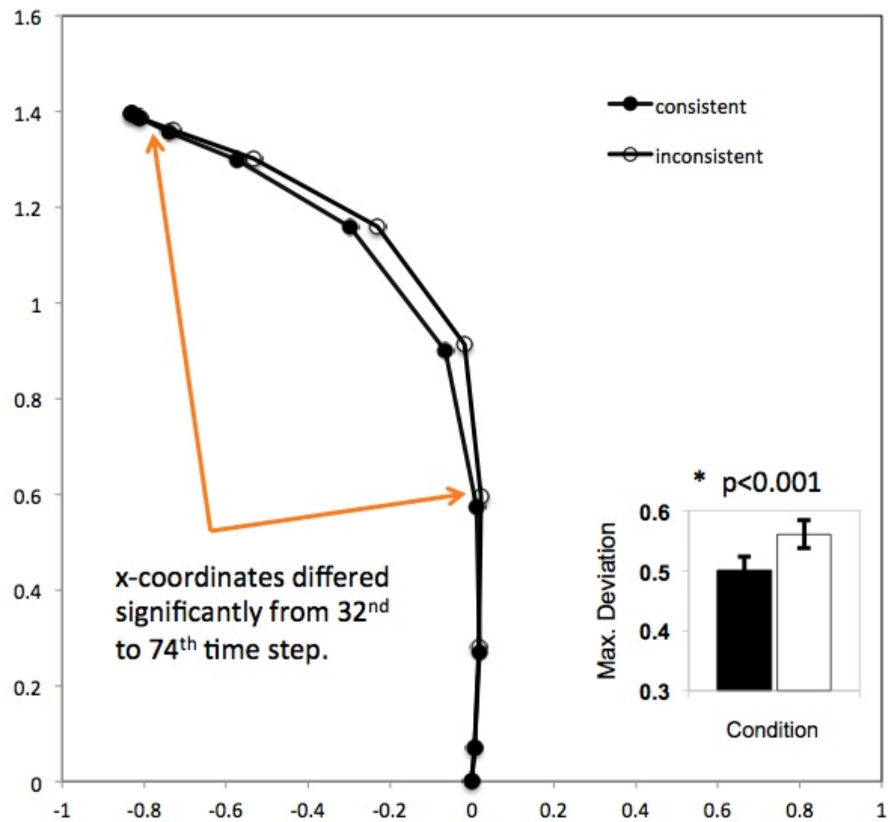

Figure 2. Mean hand trajectories as a function of consistency condition. The bar graph shows trajectories maximum deviation from the ideal, straight-line trajectory as a function of consistency condition (error bars represent standard errors of the mean)

tion could instead explain the data. It could be the case that the smooth, continuous attraction we are seeing is rather the result of averaging across trials (Spivey et al., 2005; Freeman & Dale, 2012). For example, if some trials showed zero attraction (i.e., the participants' hands moved directly toward the correct answer) and other trials were sharply deflected midflight after realization of an error, the appearance of the average trajectories would be smooth even though the cognitive processes involved were modular (that is, motor responses were not initiated until the decision was made). In this case, if we were to look at the distribution of the maximum deviation values, it would be distinctly bimodal; some of the values would be small (indicating direct trajectories) and others would be large (reflecting the midflight correction of an initially incorrect response).

To test whether this was the case, I performed a distributional analysis on the collection of maximum deviation values across all trials. Figure 3 depicts the distribution of these maximum deviation values for both consistent and inconsistent trials.

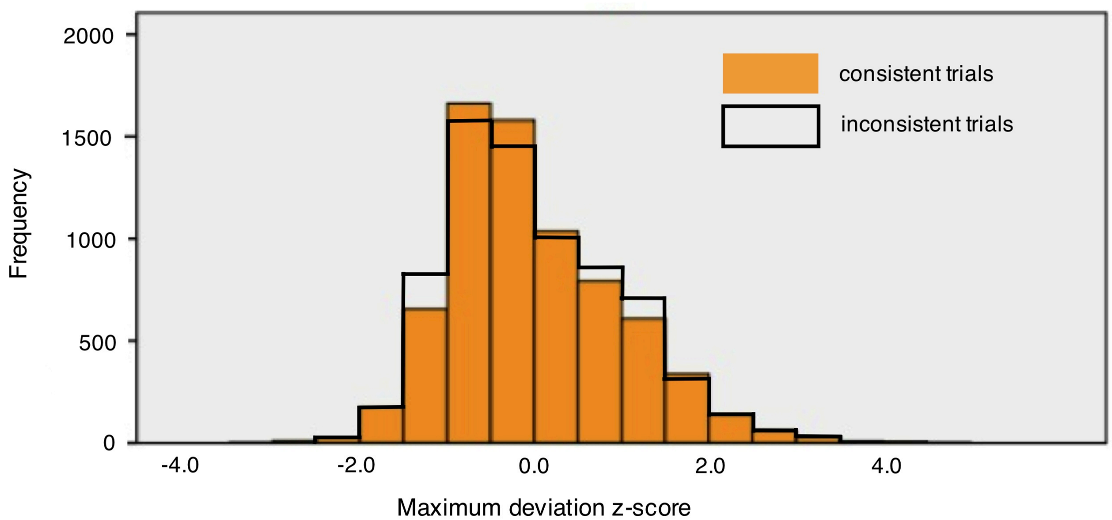

Figure 3. z-distribution of maximum deviations across 28,541 correct trials as a function of consistency between response side and numerical size.

Notice, critically, that the distribution of inconsistent trials does not differ in shape from the consistent trials, nor does it appear bimodal. Further, I computed the bimodality coefficient (SAS Institute Inc. 2012) for this distribution to be 0.423, which is less than the minimum value of 0.555 that would represent a bimodal distribution. In addition, I assessed bimodality by computing Hartigan's dip statistic D (Hartigan & Hartigan, 1985). The advantage of this statistic is that it is inferential; if $p < 0.05$, the distribution is considered to be multimodal (Freeman & Dale, 2012). Using the R package diptest (Maechler, 2013), I computed $D = 0.0018$, $p > 0.99$, confirming that the distribution is not bimodal. Taken together, these data confirm that the distribution of maximum deviation values is not bimodal, and that the smooth, continuous attraction away from the correct answer in the inconsistent trials is not the result of participants' quickly correcting their fast, incorrect initial responses.

Discussion

I found that numbers in a parity task triggered competing representations belonging to opposite response categories. This was supported by evidence indicating that when numerical magnitude was inconsistent with the side of the screen on which the correct response label was presented (e.g., a large odd number when "odd" was presented on the left side of the screen), there was a continuous deflection toward the incorrect response alternative that eventually settled before the correct response was chosen. That is, both response alternatives were activated in parallel before the correct response was chosen through a dynamic, winner-take-all process. This directly supports the hypothesis that numerical decisions occur within a continuous, competitive cognition framework (Gevers et al., 2006).

Furthermore, the present data provide a conceptual replication of the results of Santens et al. (2011) with two additions. First, this is the first study to investigate the continuous dynamics of numerical representations with a parity task (but see Gevers et al., 2010). Second, and most importantly, in this study I explicitly test the possibility that the curved trajectories that were present in Santens et al. (2011) could have resulted from averaging across trials and do not really reflect the dynamic convergence of competitive representations. Through bimodality analysis, I showed that this is not the case: the present data could not have resulted from the averaging of different types of responses, but rather, from the competition and gradual settling of partially active representations.

As a related alternative explanation, it may be the case that since participants are quickly moving their hands forward at the beginning of a trial and then making a decision while the hand is in motion, the different trajectories simply reflect that the decision in the inconsistent trials takes longer to complete. This explanation certainly fits with the presented RT data and would not differentiate between stage-based and continuous models. However, one should keep in mind that the presented trajectory data is normalized before analysis so that all trajectories are the same length (101 time steps), so any difference in RTs between trials is erased. Hence, the comparison between trajectories reflects the dynamic development of hand position in trials and not the raw time course of the hand movements. While I believe the normalization procedure rules out this alternative explanation, this is something that should be explicitly tested in future studies.

The computational model of Gevers et al. (2006) views the parity decision as a dynamic accumulation of activations for the response options EVEN and ODD. In the inconsistent condition (e.g., 2 is presented but EVEN is on the right side of the screen), the model would predict a competition that stems from the transitive juxtaposition of two immediate visuo-spatial representations; a magnitude representation (e.g, 2 = small), and a spatial representation (e.g., small = left). The natural result of these two representations is an equating of the number 2 with the left side of the screen (i.e., the SNARC effect). The claim of Santens et al. (2011) and the present study is that this competition manifests itself as a trajectory *signature* that appears as attraction toward the incorrect alternative. It is important to note that it is not necessarily the case that the hand is attracted toward the response ODD, but rather that the hand is attracted toward the left side of the screen because of the spatial-numerical pairing of 2/small with "left-side."

More generally, these data suggest that a continuous cognition framework (Spivey, 2007) may be beneficial for attempting to understand a variety of psychological puzzles. Indeed, this approach has already proved fruitful in a variety of fields, ranging among language comprehension (Spivey et al., 2005), stereotype formation (Freeman & Ambady, 2009; Freeman, Pauker, Apfelbaum, & Ambady, 2010), and semantic categorization (Dale, Kehoe, & Spivey, 2007).

Taken as a whole, the results of the present study support the burgeoning body of evidence that indicates that numerical decisions may not take place independently from our bodily affordances. By studying hand movements in even the most simple numerical task (a parity judgement task), we have revealed that numerical decisions happen within a continuous, dynamic system of partially activated cognitive states that would not be possible with the modular, computer-based metaphor of mind.

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