

# Testing a direct mapping versus competition account of response dynamics in number comparison

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Tracking continuous hand movements during number tasks has become a powerful method for disentangling competing models of numerical representation. In two experiments, participants used a computer mouse to choose whether presented numbers were greater than or less than 5. In Experiment 1, trajectories became more curved toward the incorrect response label as target numbers approached the comparison standard 5, indicating increasing response competition. However, trajectories showed a rightward bias modulated by numerical distance and target size, which supports a direct mapping account between hand movement and an ordered, spatial representation of number. In Experiment 2, I changed the direction of computer mouse movements from the standard bottom-to-top orientation to a left-to-right movement. Trajectories again became more curved toward the incorrect response label as targets approached 5, but this time, there was no modulation of trajectory bias by target size or distance. The results call into question a purely spatial direct mapping account and instead lend support to a competition-based model of response dynamics in number comparison.

*Keywords:* Numerical comparison; response competition; computer mouse tracking

Behind the appearance of formalism, the roots of mathematical thinking are intimately tied to the interaction between body and space (Lakoff & Núñez, 2000). Advanced mathematical ideas, such as limits and continuity, are often conceptualized in terms of imagined motion along a curve. In practice, expert mathematicians will often communicate new ideas via an intuitive argument, temporarily foregoing a formal, abstract proof in favor of what is often called a “hand-waving” argument. Even in the act of composing a formal proof in advanced mathematics, expert mathematicians will use gestures to communicate their ideas (Marghetis & Núñez, 2013).

Whereas gesture and hand movements seem to be an essential part of advanced mathematical thinking, they are also intimately tied to our most basic notions of number. Indeed,

much research in numerical cognition has centered around explaining the classical SNARC effect (Spatial-Numerical Association of Response Codes; Dehaene, Bossini, & Giraux, 1993), which is the finding that participants are faster to respond leftward to smaller numbers and rightward to larger numbers. Though the SNARC effect has been replicated in a wide variety of contexts and experimental settings (Hubbard, Piazza, Pinel, & Dehaene, 2005), there are still open questions about its origin.

One of the more widely-accepted models of the SNARC effect is based on the idea of a mental number line (Restle, 1970; Dehaene et al., 1993) and is known as the “direct mapping account” (Santens & Gevers, 2008; Ishihara et al., 2006; Schiller, Eloka, & Franz, 2016). In the direct mapping account, mental representations of symbolic numbers are hypothesized to be arranged on a spatially-oriented mental number line, where magnitude increases in the same direction as reading direction (Shaki, Fischer, & Petrusic, 2009). These numbers are then mapped to external space in a one-to-one fashion (Ishihara et al., 2006) so that relative position on the number line is preserved in external space. Hence, when participants are asked to use a leftward response to answer a question about a large number (or vice versa), there is a mismatch between the response location (left) and the external projection of the represented number (right). This mismatch explains the relative slowdown that participants exhibit when using leftward responses for large numbers and rightward responses for small numbers.

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and Fias (2006) proposed a fundamentally different model that is based on response competition. In their model, Gevers et al. (2006) present a three-layer neural network, where the input layer consists of a numerical representation and the output layer consists of nodes representing response alternatives (e.g., leftward or rightward). In addition, there is a hidden layer that is responsible for a task-dependent categorical representation of numbers (e.g., even/odd for a parity task, small/large for a magnitude task). Response times are then modeled by the number of network cycles required for the correct response node to “win” by reaching a certain activation threshold. Critically, the SNARC effect is modeled as a competition effect, where activations in competing response nodes rise and fall over time.

Though much research has attempted to understand these associations between numerical and spatial thinking, fewer studies have focused on the dynamics of the actual hand movements that occur during numerical thought. It is perhaps surprising that these small hand movements can provide a direct behavioral signature of the types of mental representations that are formed while thinking about numbers. Tracking these hand movements has become a popular method for testing various accounts of phenomena in numerical cognition (Fischer & Hartmann, 2014; Faulkenberry & Rey, 2014). As such movements reflect a projection of internal cognitive processes onto an observable behavioral output (Spivey, Grosjean, & Knoblich, 2005; Spivey, 2007), hand tracking provides a window into the internal processes that evolve during a numerical decision.

A number of recent numerical cognition studies have exploited this technique in various forms, such as tracking the hand (Song & Nakayama, 2008), tracking the finger (Santens, Goossens, & Verguts, 2011; Dotan & Dehaene, 2013), and tracking the computer mouse (Marghetis, Núñez, & Bergen, 2014; Faulkenberry, 2014; Ganor-Stern & Goldman, 2014; Faulkenberry, Montgomery, & Tennes, 2015; Haslbeck, Wood, & Witte, 2015; Faulkenberry, Cruise, Lavro, & Shaki, 2016). Among the first of this line of studies was Song and Nakayama (2008), who tracked participants’ hand movements as they quickly judged whether a single digit number was less than or greater than 5 by pointing to the left or right side of a computer screen. Song and Nakayama found that movement trajectories became more curved toward the center of the screen as the presented number approached 5. Song and Nakayama (2008) explained this pattern in terms of the direct mapping account, positing a direct correspondence between the position of a manual response and the position of a number on a mental number line.

Santens et al. (2011) critiqued this result on methodological grounds and offered an alternative to the direct mapping account. Santens et al. (2011) argued that since Song and Nakayama (2008) used only one response rule throughout their experiment (less than 5, move left; greater than 5, move

right), their results could also be explained by a competition-based model of small number representation (Verguts, Fias, & Stevens, 2005; Gevers et al., 2006), thus requiring no assumption of a spatial correspondence between responses and a mental number line. To test between these two accounts, Santens et al. (2011) used two response rules. One response rule used a number line congruent response mapping identical to that of Song and Nakayama (2008), where participants moved their finger leftward for a “less than” decision and rightward for a “greater than” decision. The other response rule used a number line incongruent response mapping, where the “less than” decision required rightward movement and the “greater than” decision required leftward movement.

To see how the two models differ in their predictions, consider Figure 1, which provides a schematic representation of the idealized predicted trajectories for targets smaller than 5. Regardless of response mapping, the direct mapping account predicts that trajectories for the stimulus 4 will lie to the right of trajectories for the stimulus 1. This is because the direct mapping account hypothesizes a direct, one-to-one correspondence between the number’s position on a mental number line and the response location in physical space. A similar picture can be seen in Figure 2, where the direct mapping account predicts that trajectories for the stimulus 6 will lie to the left of trajectories for the stimulus 9. Note that the straight lines depicted in the predictions of the direct mapping account simply represent a relatively direct path toward the response location with less curvature, not necessarily an absolutely straight trajectory with zero curvature.

Whereas the direct mapping account (Song & Nakayama, 2008) and the competition account (Verguts et al., 2005; Gevers et al., 2006; Santens et al., 2011) make identical predictions for trajectories in the number line congruent response mapping, their predictions are in opposition for the incongruent response mapping. As indicated by the dashed lines in Figures 1 and 2, the direct mapping account predicts that as numerical distance increases, trajectories in the incongruent response mapping become more curved. On the other hand, the competition account predicts that these trajectories will become less curved (due to reduced competition as distance increases). Santens et al. (2011) found exactly this result; curvatures (as indexed via horizontal deviation from the vertical midline) decreased as a function of the target’s increasing distance from 5. As such, Santens et al. (2011) interpreted their results as support for the competition account.

One limitation to the conclusions of Santens et al. (2011) is that they did not consider that average trajectories can potentially hide two very different types of hand movements (Freeman & Dale, 2012). It is possible that a curved trajectory of the type seen in Figures 1 and 2 can result from averaging across two types of trajectories; one trajectory with little curvature that moves directly to the correct answer,

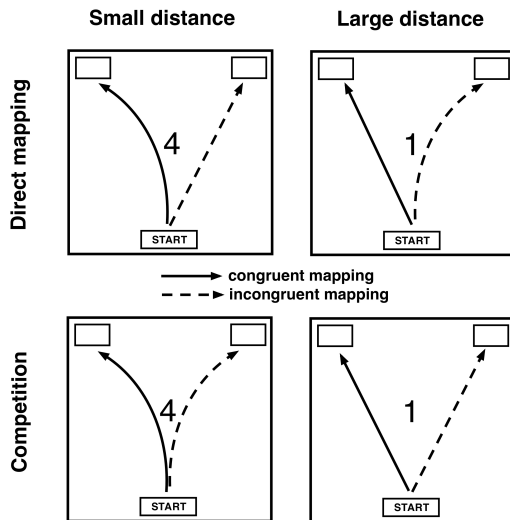


Figure 1. Idealized predicted trajectories toward small targets (less than 5) from the direct mapping account (Song & Nakayama, 2008) and the competition account (Verguts, Fias, & Stevens, 2005), displayed as a function of numerical distance (small versus large) and response mapping (congruent versus incongruent). Figure is adapted from Santens, Goossens, and Verguts (2011).

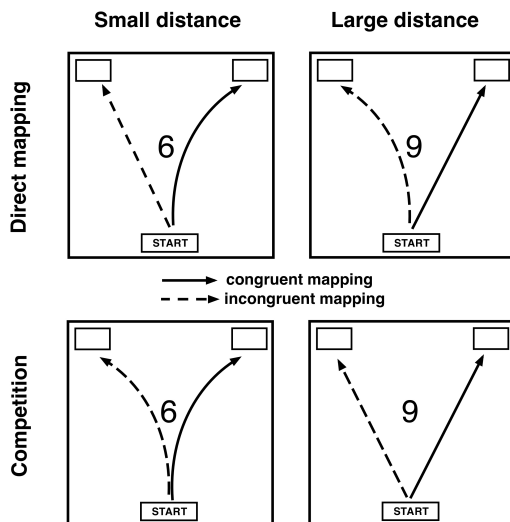


Figure 2. Idealized predicted trajectories toward large targets (greater than 5) from the direct mapping account (Song & Nakayama, 2008) and the competition account (Verguts, Fias, & Stevens, 2005), displayed as a function of numerical distance (small versus large) and response mapping (congruent versus incongruent). Figure is adapted from Santens, Goossens, and Verguts (2011).

and another path that begins in the wrong direction but is quickly corrected in midflight. Concretely, behavior of this type could produce the curved trajectory represented by the dashed line in the bottom left of each of Figures 1 and 2, thus appearing to support the competition account. However, the underlying cognitive processes would be more in line with a direct mapping account. Hence, the conclusions of Santens et al. (2011) could be a bit premature.

Note, however, that one should be able to detect such behaviors simply by analyzing the distribution of mouse trajectories. These disparate cognitive processes would be reflected by a bimodal distribution of response trajectories (Freeman & Dale, 2012). Thus, to conclude that a curved trajectory indeed is representative of competition between parallel and partially active response options, one must take care to rule out this alternative by assessing whether the distribution of trajectory curvatures is indeed bimodal.

In the present study, I used computer mouse tracking to test between the direct mapping and competition accounts of the dynamics of number comparison (e.g., Song & Nakayama, 2008; Santens et al., 2011). Specifically, I also analyzed the distributions of trajectories in order to rule out an alternative explanation of the results of Santens et al. (2011). In addition, I aimed to perform a novel test to further tease apart the direct mapping and competition accounts by computing the amount of trajectory asymmetry between congruent and incongruent trials. As one can see in Figure 1, the direct mapping account predicts that trajectories for small targets would be biased toward the right for small distance trials and toward the left for large distance trials. On the other hand, Figure 2 shows the opposite pattern; namely that trajectories for large targets would be biased toward the left for small distance trials and to the right for large distance trials. The competition account makes a different prediction. Since the competition account models trajectory curvature as depending only on the numerical distance between the target and the comparison, and not on the specific response mapping, the competition account would predict no trajectory bias as a function of either distance or target size.

## Experiment 1

The purpose of Experiment 1 was to replicate the study of Santens et al. (2011) using computer mouse tracking and test against an alternative explanation. In addition, I aimed to test for asymmetry between trajectories for congruent and incongruent trials, which is a novel and previously untested signature for the direct mapping account (Song & Nakayama, 2008).

## Method

**Participants.** Sixty-four undergraduate students (52 female, mean age = 24.1 years, age range 18 to 57) participated in this experiment in exchange for partial course credit in

their psychology courses. Eight participants reported being left hand-dominant, but all reported that they used their right hand for the computer mouse. The experiment was reviewed and approved by the institutional review board at Tarleton State University.

**Apparatus.** The experiment was implemented using the MouseTracker software package (Freeman & Ambady, 2010). The experimental trials were presented on a 20 inch iMac desktop computer with a screen resolution of 1,280 x 1,024 pixels and a refresh rate of 60 Hz. Input was captured via a Dell optical mouse connected via USB. Participants were seated approximately 60 cm from the computer and allowed to hold the computer mouse in a position which was comfortable. All participants held the mouse in the right hand, positioned slightly to the right of center on the computer table. We ran the MouseTracker program on the iMac using a virtual Windows XP environment via Parallels. Following the recommendations of Fischer and Hartmann (2014), we disabled the “dynamic acceleration” option and lowered the speed of the mouse movements on the screen to the second-lowest possible speed in the mouse settings dialog. This is done to prevent quick and erratic mouse movements, resulting in a smooth and more reliable record of participants’ hand movements. The resulting displacement ratio of the mouse to screen movement was 1 cm to 100 pixels.

**Stimuli and procedure.** Participants were asked to perform a number comparison task using the digits 1 through 9, excluding 5, which we used as the comparison standard on each trial. Participants were told that on every trial a number would appear in the center of the screen, and they would be asked to choose, as quickly as possible, whether the number was less than 5 or greater than 5. Each trial started with a blank screen presented for 1000 ms, followed by a screen that displayed the response labels SMALLER and LARGER at the top left and right of the screen, respectively. Each response label was presented in Arial font with point size 24.

The order of the response labels was switched once midway through the experiment; half of the participants started with the SMALLER-LARGER ordering, while the other half began with the LARGER-SMALLER ordering. After 1000 ms, a START button appeared. Once the START button was pressed, one of the stimulus numerals appeared in the center of the screen, presented in Arial font with point size 48. Participants then clicked on the correct of these two options; while doing so, the software recorded the streaming ( $x, y$ ) coordinates of the computer mouse approximately 63 times per second.

We manipulated the spatial congruity of the response labels SMALLER and LARGER: in the congruent condition, SMALLER appeared in the upper left corner and LARGER appeared in the upper right corner. In the incongruent condition, these labels were reversed. In half of the trials, the correct answer was on the left side, whereas on the other half

of the trials, the correct answer was on the right side.

For incorrect responses, the program displayed an “X” for 1000 ms. To ensure that trajectories reflected online processing, participants were encouraged to begin their movements as early as possible and were warned if initiated movement later than 250 ms following number pair presentation. This instruction is customarily included in mousetracking studies so that trajectories reflect the dynamics of a decision process rather than simply reflecting the kinematics of a response choice after the choice has already been made (Freeman & Ambady, 2009; Spivey et al., 2005).

For each of the two counterbalanced spatial congruity conditions, participants completed two blocks of 160 trials (20 repetitions of each stimulus number, randomly presented), with a short break in between each block. In all, each participant completed 640 experimental trials in a single 45 minute session.

## Results

Participants completed a total of 40,960 trials. Of these, 84 trials contained a response error (0.20%). From these trials, we excluded an additional 659 trials for which overall reaction time exceeded 3 standard deviations from the mean reaction time across all correct trials (1.6%). All further analyses were conducted on the remaining 40,217 trials. All statistical analyses were performed using the R statistical package (R Development Core Team, 2011). All figures shown were created in R using the ggplot2 package (Wickham, 2009).

**Time analyses.** For each trial, the MouseTracker software recorded two time-based performance measures: reaction time (RT), the total time elapsed between clicking the START button and the target mouse click; and initiation time (Init), the time elapsed between target onset after clicking the START button and the onset of mouse movement. From these two measures, I calculated movement time (MT), the actual duration of mouse movement, via the relationship  $MT = RT - \text{Init}$ .

Movement times were submitted to a 2 (Response mapping condition: congruent vs. incongruent) x 2 (Response side: leftward vs. rightward) x 4 (Distance: 1, 2, 3, 4) repeated measures analysis of variance (see Table 1). There was a significant main effect of distance,  $F(3, 189) = 94.58$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.60$ . As can be seen in Figure 3, movement times decreased with increasing numerical distance. To confirm this, I tested whether movement time varied as a linear function of numerical distance by performing a linear contrast, which was significant,  $t = -4.42$ ,  $p < 0.001$ . There was also a significant main effect of response side,  $F(1, 63) = 6.32$ ,  $p = 0.01$ ,  $\eta_p^2 = 0.09$ . Leftward responses were 14 ms faster than rightward responses. Finally, there was a significant interaction between response mapping condition and response side,  $F(1, 63) = 5.10$ ,  $p = 0.03$ ,  $\eta_p^2 = 0.07$ . As Figure 3 shows, leftward responses with an in-

congruent response mapping (LARGER - SMALLER) were significantly slower than leftward responses with a congruent response mapping (SMALLER - LARGER). However, this effect disappeared for rightward responses. No other effects in the ANOVA model were significant (all  $F$  - ratios less than 0.98).

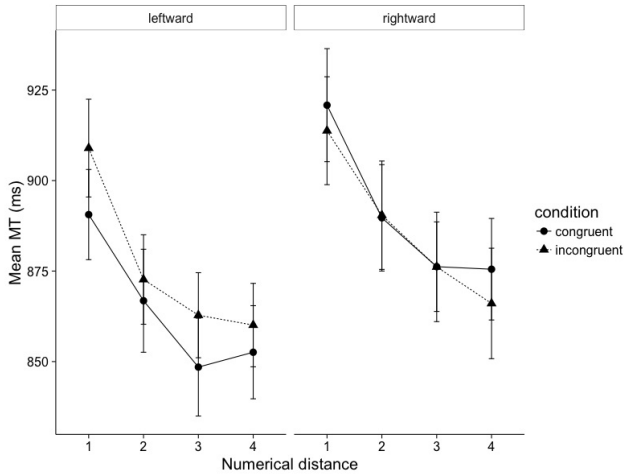


Figure 3. Mean movement times in Experiment 1 as a function of numerical distance (1, 2, 3, 4), spatial congruity (congruent versus incongruent), and response side (leftward versus rightward). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

Similarly, initiation times were submitted to a 2 (Response mapping condition: congruent vs. incongruent)  $\times$  2 (Response side: leftward vs. rightward)  $\times$  4 (Distance: 1, 2, 3, 4) repeated measures analysis of variance (see Table 1). Initiation times did not differ between conditions on any factor (all  $F$  ratios less than 3.1). Such results indicate that the effects of numerical distance and response mapping on numerical representations may continue to bias action well into the response phase (e.g., Buc Calderon, Verguts, & Gevers, 2015; Faulkenberry et al., 2016).

**Trajectory analyses.** As in other recent mouse tracking studies, hand trajectories were measured by recording the streaming  $x, y$  - coordinates of the computer mouse during each trial. Raw mouse trajectories were pre-processed in MouseTracker (Freeman & Ambady, 2010) so that all hand trajectory data was rescaled onto a standard coordinate space of  $[-1, 1] \times [0, 1.5]$ . All trajectories were normalized via linear interpolation to consist of exactly 101 timesteps. This step is customarily done to allow direct comparison of trajectories in different conditions without the confounding effect of differing movement times. Although there is some debate about whether averaging over length-normalized trajectories leads to systematic bias in the mean trajectories (Poli & Salvaris, 2011), such preprocessing was necessary to test the predictions of this experiment. Specifically, the normaliza-

tion procedure allowed for direct comparison of trajectory curvatures, indexed via area under the curve (AUC). Since MouseTracker computes AUC for each trial by summing the areas of 100 trapezoids that comprise the area between the trajectory path and the ideal straight line between the start box and the response box, it is essential that the trajectories each be comprised of the same number of timesteps.

Average mouse trajectories as a function of response mapping and response side are depicted in Figure 4. To test how curvatures changed as a function of numerical distance, I used area under the curve (AUC) as an index of the trajectory curvature on each trajectory. Mean AUC values can be seen in Table 1. AUC values were submitted to a 2 (Response mapping condition: congruent vs. incongruent)  $\times$  2 (Response side: leftward vs. rightward)  $\times$  4 (Distance: 1, 2, 3, 4) repeated measures analysis of variance. There was a significant main effect of numerical distance,  $F(3, 189) = 105.2$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.63$ . As can be seen in Figure 5, AUC decreased as numerical distance increases (linear contrast;  $t = -6.12$ ,  $p < 0.001$ ). There was also a significant main effect of response mapping condition,  $F(1, 63) = 16.46$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.21$ . Trajectories from the incongruent response mapping had more curvature than those from the congruent response mapping.

There was a small but statistically significant interaction between response mapping condition and response side,  $F(1, 63) = 4.12$ ,  $p < 0.05$ ,  $\eta_p^2 = 0.06$ , where the effect of response mapping condition was greater for leftward trajectories than for rightward trajectories. Finally, there was a significant interaction between numerical distance and response mapping condition,  $F(3, 189) = 12.15$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.16$ . Whereas the incongruent response mapping generally resulted in greater curvatures across all distances, this pattern did not hold for rightward trajectories (see Figure 5). This claim is qualified via the significant three-way interaction between distance, response mapping condition, and response side,  $F(3, 189) = 4.74$ ,  $p = 0.003$ ,  $\eta_p^2 = 0.07$ .

The critical test to tease apart whether trajectory data lend better support to the direct mapping account or the competition account comes from the pattern of curvatures from incongruent trials. Specifically, the direct mapping account predicts that curvature on incongruent trials increases with numerical distance, whereas the competition account predicts the opposite (Santens et al., 2011). The present data indeed shows a linear decrease in curvatures with increasing numerical distance (see Figure 5). However, the presence of an interaction between distance and response mapping condition indicates that this general decreasing pattern may be different between congruent and incongruent trials. So, to test between these two competing models, I tested incongruent trials in a separate analysis.

To this end, AUC values on incongruent trials were submitted to a 2 (Response side: leftward vs. rightward)  $\times$  4

Table 1  
*Mean (SD) of performance measures for trajectories in Experiment 1*

Distance	Congruent Trials				Incongruent Trials			
	1	2	3	4	1	2	3	4
Leftward trajectories								
MT (msec)	897 (42)	873 (43)	853 (42)	855 (40)	909 (44)	874 (44)	868 (42)	865 (38)
Init (msec)	84 (15)	83 (15)	83 (12)	83 (16)	87 (13)	88 (16)	89 (14)	89 (15)
AUC	0.95 (.35)	0.83 (.35)	0.64 (.29)	0.69 (.34)	1.14 (.46)	0.92 (.46)	0.89 (.42)	0.85 (.41)
Rightward trajectories								
MT (msec)	912 (48)	894 (49)	876 (39)	877 (41)	915 (43)	890 (43)	872 (47)	871 (46)
Init (msec)	84 (11)	84 (15)	83 (13)	84 (17)	90 (13)	87 (14)	86 (15)	88 (14)
AUC	0.81 (.39)	0.83 (.32)	0.64 (.33)	0.61 (.33)	0.98 (.39)	0.75 (.38)	0.67 (.37)	0.68 (.44)

Note. MT = movement time, Init = initiation time, AUC = area under curve.

(Distance: 1, 2, 3, 4) repeated measures analysis of variance. As in the original analysis, there was a significant main effect of numerical distance,  $F(3, 189) = 36.71$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.37$ , which indicated that curvature decreased as a function of numerical distance (linear contrast;  $t = -3.83$ ,  $p < 0.001$ ). Critically, there was no interaction between numerical distance and response side ( $F < 0.35$ ), indicating that curvature decreased with increasing numerical distance regardless of response side. This result is only predicted by the competition model (Santens et al., 2011), which states that as numerical distance increases, response competition decreases, leading to straighter, less curved trajectories.

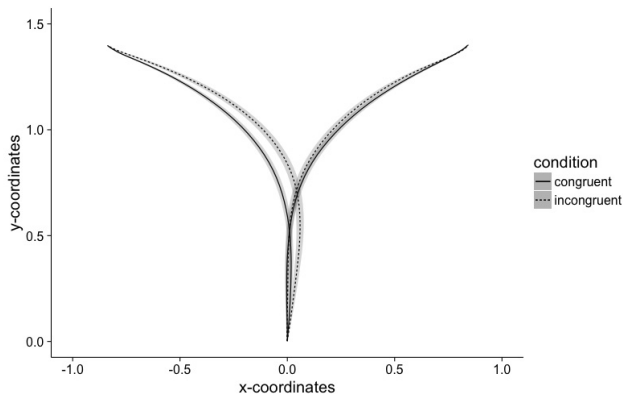


Figure 4. Average computer mouse trajectories in Experiment 1 as a function of spatial congruity condition (congruent versus incongruent) and response side (leftward versus rightward). Shading represents one standard error, computed from the mean  $x$ -coordinates of trajectories over the sample of 64 participants.

**Distribution of trajectories.** The trajectory data above lend tentative support to the competition model of numerical representation (Verguts et al., 2005), and hence reflect the view that during the numerical decision process, participants

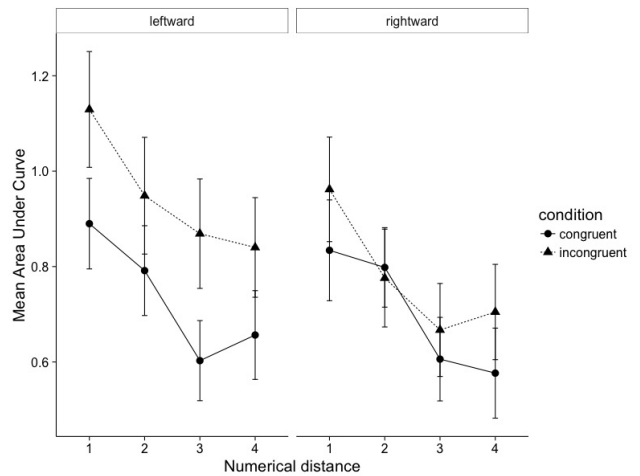


Figure 5. Mean area under the curve (AUC) in Experiment 1 as a function of numerical distance (1, 2, 3, 4), spatial congruity condition (congruent versus incongruent), and response side (leftward versus rightward). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

formed partially-active representations of both response alternatives until the winning representation was stabilized and the correct answer was chosen. However, there is an alternative explanation that could explain the data. It could be the case that the smooth, continuous attraction we see in Figure 4 is simply the result of averaging across fundamentally different types of trials (Spivey et al., 2005; Freeman & Dale, 2012). For example, if some trials showed no attraction toward the competitor (i.e., the participants' hands moved directly toward the correct answer) and other trials were sharply deflected midflight as a corrective motion after an initial error, the appearance of the average trajectories would be smooth, even though the cognitive processes

involved were modular (that is, motor responses were not initiated until the decision was made). In this case the distribution of the AUC values would be bimodal; some of the values would be small (indicating direct trajectories) and others would be large (reflecting the midflight correction of an initially incorrect response).

To test against this possibility, I analyzed the distribution of AUC values for indications of bimodality (see Figure 6). Specifically, I transformed AUC values to  $z$ -scores by participant, then computed a bimodality coefficient (SAS Institute Inc., 2012) for the distribution of AUC  $z$ -scores for incongruent trials. This bimodality coefficient was 0.412, which is less than the minimum value of 0.555 that would represent a bimodal distribution. In addition, I tested bimodality by computing Hartigan’s dip statistic  $D$  (J. A. Hartigan & P. M. Hartigan, 1985).  $D$  is an inferential statistic; if  $p < 0.05$ , the distribution is considered to be multimodal (Freeman & Dale, 2012). Using the R package `diptest` (Maechler, 2013), I computed  $D = 0.0009$ ,  $p > 0.99$ , confirming that the distribution is not bimodal. Both results confirm that the distribution of AUC values is not bimodal, and that the smooth, continuous attraction away from the correct answer in the incongruent trials is not the result of participants’ quickly correcting their fast, incorrect initial responses.

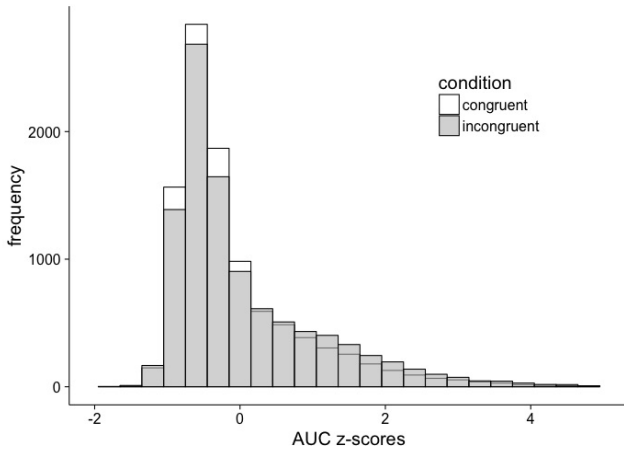


Figure 6. Distribution of  $z$ -scores of AUC values in Experiment 1 as a function of spatial congruity condition (congruent versus incongruent).

### Testing asymmetry between response mappings.

While the trajectory data above seem to indicate support for the competition model over the direct mapping model, the presence of interactions between numerical distance, response mapping condition, and response side prevent a clean, direct interpretation. Thus, I decided to supplement the AUC analysis with a novel analysis based on symmetry of trajectories. A direct inspection of Figures 1 and 2 reveals that one main difference between the direct mapping and competition accounts lies in the symmetry between congruent and incongruent response mappings.

Specifically, the direct mapping account predicts that trajectories are asymmetric about the vertical movement axis. For example, in Figure 1 we see that for the stimulus 1, the trajectory toward the upper left response box (a congruent response mapping) has very little curvature, but the trajectory toward the upper right response box (an incongruent response mapping) has a fair bit of curvature. Hence, taken as a pair of trajectories, this pair is quite asymmetric. On the other hand, the competition account predicts that the same trajectory pairs would be symmetric about the vertical movement axis.

To test for asymmetry between congruent and incongruent response mappings, I first computed an asymmetry coefficient  $\mathcal{A}$  for each participant. Intuitively,  $\mathcal{A}$  represents the average amount of asymmetry across the average trajectories for both response mappings (congruent and incongruent) over the 101 normalized timesteps. Specifically, it is computed as:

$$\mathcal{A} = \frac{1}{101} \left[ \sum_{i=1}^{101} x_C(i) + x_I(i) \right]$$

where  $x_C(i)$  is the mean  $x$ -coordinate at timestep  $i$  across all congruent response trials, and  $x_I(i)$  is the mean  $x$ -coordinate at timestep  $i$  across all incongruent response trials. Notice that by definition,  $\mathcal{A} > 0$  implies an asymmetry characterized by rightward bias and  $\mathcal{A} < 0$  implies a leftward bias, whereas  $\mathcal{A} = 0$  implies symmetric response trajectories.

Using this formulation, one can derive predictions from the direct mapping and competition accounts. Specifically, for small targets, the direct mapping account predicts that as numerical distance increases,  $\mathcal{A}$  decreases from positive to negative. For large targets, the direct mapping account predicts that as distance increases,  $\mathcal{A}$  increases from negative to positive. In other words, a key signature of the direct mapping account would be an interaction between target size (smaller versus larger) and numerical distance (small versus large).

To test this, I submitted asymmetry scores  $\mathcal{A}$  to a 2 (target size: smaller than 5, larger than 5)  $\times$  2 (numerical distance: small, large) repeated measures analysis of variance. As can be seen in Figure 7, there was a significant main effect of target decision,  $F(1, 63) = 14.38$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.19$ . Trajectories toward the “larger than 5” decision exhibited more rightward bias than trajectories toward the “smaller than 5” decision. There was no main effect of distance on asymmetry ( $F < 0.43$ ). There was a small, but statistically significant interaction between target decision and distance,  $F(1, 63) = 4.15$ ,  $p < 0.05$ ,  $\eta_p^2 = 0.06$ . For large targets, the asymmetry increased with numerical distance, whereas the asymmetry decreased for small targets.

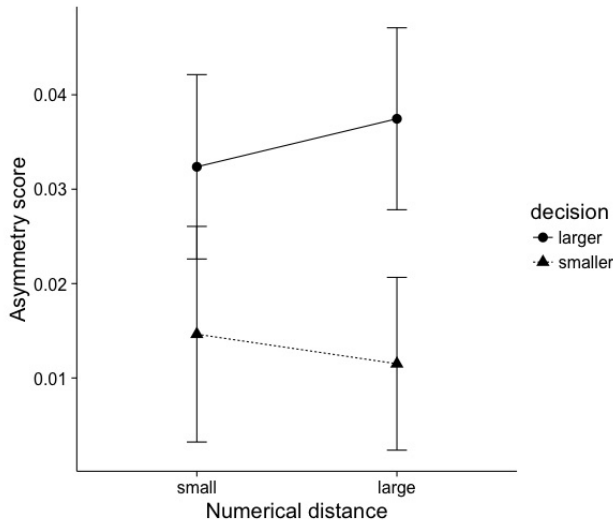


Figure 7. Mean asymmetry score in Experiment 1 as a function of target decision (smaller versus larger) and numerical distance (small versus large). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

## Discussion

The results of Experiment 1 were mixed. On one hand, the curvature data (as indexed via trial-by-trial AUC measures) indicated support for the competition account (Santens et al., 2011). Curvatures decreased as a function of numerical distance for incongruent trials, which cannot be directly explained by the direct mapping account. On the other hand, computing trajectory asymmetry between congruent and incongruent trials led to a different picture. Specifically, the asymmetry scores revealed a target decision by distance interaction, where asymmetry increased with distance for large targets but decreased with distance for small targets. This result cannot be explained by a competition account, which predicts no asymmetry between congruent and incongruent trials. As such, what seems like clear evidence for the competition account is obscured by a pervasive direct mapping signature in the asymmetry scores.

However, it is possible that this direct mapping signature stems from biomechanical constraints placed upon participants from apparatus and experimental design. Note that all participants held the computer mouse in the right hand. When this biomechanical limitation is paired with a left-right orientation of target responses, rightward bias may be the natural result. If this is the case, then one cannot directly interpret these results (particularly the asymmetry analysis) as support for either model.

To overcome this limitation, I performed a second experiment where participants performed an identical magnitude comparison task, but instead of the usual movement trajec-

tory of starting in the bottom center of the screen and moving either to the upper left or right, participants began each trial in the vertical center of the left side of the screen and moved rightward to either the upper right or the lower right.

## Experiment 2

The purpose of Experiment 2 was to investigate whether the direct mapping signature revealed in asymmetry scores in Experiment 1 was a result of biomechanical constraints that result from the experimental setup. Specifically, all participants used their right hand to hold the computer mouse, so rightward motion may have been facilitated and leftward motion inhibited. Such movement constraints could have resulted in the rightward bias seen in Experiment 1 trajectories. On the other hand, the rightward bias might have been the result of some direct mapping between numerical representations and manual responses. To test between these two explanations, I designed the same task as in Experiment 1, but changed initial movement direction from vertical (starting at bottom of screen and moving upward toward top left or right) to horizontal (starting at left and moving rightward toward top right or bottom right).

## Method

**Participants.** Thirty-two undergraduate students (26 female, mean age = 20.9 years, age range 18 to 32) participated in this experiment in exchange for partial course credit in their psychology courses. Three participants reported being left hand-dominant, but as in Experiment 1, all reported that they used their right hand for the computer mouse. The experiment was reviewed and approved by the institutional review board at Tarleton State University.

**Apparatus.** The apparatus was identical to that used in Experiment 1.

**Stimuli and procedure.** Except for orientation of the screen and the movement parameters of the computer mouse, the stimuli and procedure were identical to Experiment 1. Each trial started with a blank screen presented for 1000 ms, followed by a screen that displayed the response labels SMALLER and LARGER at the upper right and lower right of the screen. After 1000 ms, a START button appeared, vertically centered on the far left side of the screen. Once the START button was pressed, one of the stimulus numerals appeared in the center of the screen, presented in Arial font with point size 48. The deadline for response initiation was increased to 400 ms, as an earlier pilot experiment indicated that participants had difficulty initiating left-to-right movement within the 250 ms threshold used in Experiment 1.

As in Experiment 1, we manipulated the spatial congruity of the response labels SMALLER and LARGER: in the congruent condition, SMALLER appeared in the lower right corner and LARGER appeared in the upper right corner. This



choice of orientation was based on previous work identifying a vertically oriented mapping of small numbers to lower space and large numbers to upper space (e.g., Ito & Hatta, 2004; Schwarz & Keus, 2004). In the incongruent condition, these labels were reversed. In half of the trials, the correct answer was on the upper right, whereas on the other half of the trials, the correct answer was on the lower right.

For each of the two counterbalanced spatial congruity conditions, participants completed 160 trials (20 repetitions of each stimulus number, randomly presented). In all, each participant completed 320 experimental trials in a single 30 minute session.

## Results

Participants completed a total of 10,240 trials. Of these, 40 trials contained a response error (0.39%). From these trials, I excluded an additional 189 trials for which overall reaction time exceeded 3 standard deviations from the mean reaction time across all correct trials (1.9%). All further analyses were conducted on the remaining 10,011 trials.

**Time analyses.** As in Experiment 1, the MouseTracker software recorded reaction time (RT) and initiation time (Init). From these two measures, I calculated movement time (MT) via the relationship  $MT = RT - \text{Init}$ .

Movement times were submitted to a 2 (Response mapping condition: congruent vs. incongruent)  $\times$  2 (Response direction: upward versus downward)  $\times$  4 (Distance: 1, 2, 3, 4) repeated measures analysis of variance (see Table 2). There was a significant main effect of distance,  $F(3, 93) = 28.14$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.48$ . As can be seen in Figure 8, movement times decreased as numerical distance increases, as confirmed via a significant linear contrast,  $t = -3.22$ ,  $p = 0.001$ . No other effects were significant (all  $F$  values less than 3.2).

Initiation times were submitted to a 2 (Response mapping condition: congruent vs. incongruent)  $\times$  2 (Response direction: upward versus downward)  $\times$  4 (Distance: 1, 2, 3, 4) repeated measures analysis of variance (see Table 2). As in Experiment 1, initiation times did not differ as a function of any factor (all  $F$  values less than 1.6).

**Trajectory analyses.** Average mouse trajectories as a function of response mapping condition and response side are depicted in Figure 9. AUC values were submitted to a 2 (Response mapping condition: congruent vs. incongruent)  $\times$  2 (Response direction: upward vs. downward)  $\times$  4 (Distance: 1, 2, 3, 4) repeated measures analysis of variance (see Table 2). There was a significant main effect of numerical distance,  $F(3, 93) = 26.5$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.46$ . As can be seen in Figure 10, AUC decreased as numerical distance increases (linear contrast;  $t = -3.43$ ,  $p < 0.001$ ). There was also a small, but statistically significant main effect of response mapping condition,  $F(1, 31) = 4.27$ ,  $p < 0.05$ ,  $\eta_p^2 = 0.12$ . Trajectories from the incongruent response mapping had slightly more curvature than those from the congru-

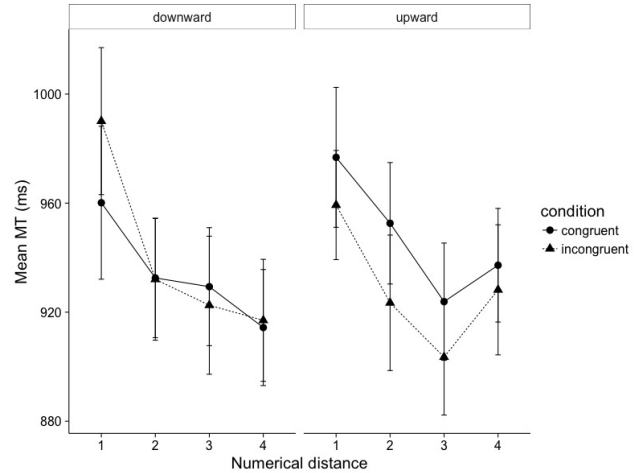


Figure 8. Mean movement times in Experiment 2 as a function of numerical distance (1, 2, 3, 4), spatial congruity condition (congruent versus incongruent), and response side (leftward versus rightward). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

ent response mapping. Finally, there was a significant main effect of response direction,  $F(1, 31) = 32.09$ ,  $p < 0.001$ ,  $\eta_p^2 = 0.51$ ; upward trajectories were significantly more curved than downward trajectories. None of the remaining interactions were statistically significant (all  $F$  values less than 2.1). Critically, there was no interaction between condition and distance, which implies that trajectories for both congruent and incongruent trials became less curved as numerical distance increases. Particularly, the decrease in curvatures on incongruent trials is only predicted by the competition model (Santens et al., 2011).

**Distributional analyses of trajectories.** As in Experiment 1, I tested against the possibility that the smooth, average trajectories we see in Figure 9 were not the result of competitive processes but rather a statistical artifact of collapsing across fundamentally different cognitive process signatures. The distribution of  $z$ -scores of AUC values can be seen in Figure 11. I computed the bimodality coefficient for the distribution of AUC  $z$ -scores for incongruent trials to be 0.229, which is less than the minimum value of 0.555 that would represent a bimodal distribution. In addition, I tested bimodality by computing Hartigan's dip statistic as  $D = 0.0030$ ,  $p > 0.99$ , confirming that the distribution is not bimodal. Both results mirror those in Experiment 1, and confirm that the distribution of AUC values is not bimodal.

**Testing asymmetry between response mappings.** As in Experiment 1, I tested for asymmetry between congruent and incongruent trajectories. Given the side-to-side orientation of mouse trajectories in Experiment 2, the computation of the asymmetry score was slightly different. Specifically,

Table 2  
*Mean (SD) of performance measures for trajectories in Experiment 2*

Distance	Congruent Trials				Incongruent Trials			
	1	2	3	4	1	2	3	4
Upward trajectories								
MT (msec)	960 (78)	933 (61)	929 (60)	914 (59)	990 (75)	932 (62)	923 (70)	917 (62)
Init (msec)	140 (42)	134 (31)	135 (23)	141 (30)	133 (34)	130 (27)	128 (30)	136 (27)
AUC	0.42 (.10)	0.39 (.12)	0.35 (.10)	0.35 (.11)	0.43 (.09)	0.41 (.09)	0.35 (.09)	0.37 (.09)
Downward trajectories								
MT (msec)	977 (71)	953 (62)	924 (60)	937 (58)	959 (56)	923 (69)	904 (59)	928 (66)
Init (msec)	138 (28)	130 (24)	140 (28)	138 (34)	136 (31)	135 (28)	128 (27)	132 (29)
AUC	0.27 (.10)	0.23 (.10)	0.23 (.08)	0.22 (.09)	0.30 (.09)	0.28 (.10)	0.25 (.09)	0.26 (.10)

Note. MT = movement time, Init = initiation time, AUC = area under curve.

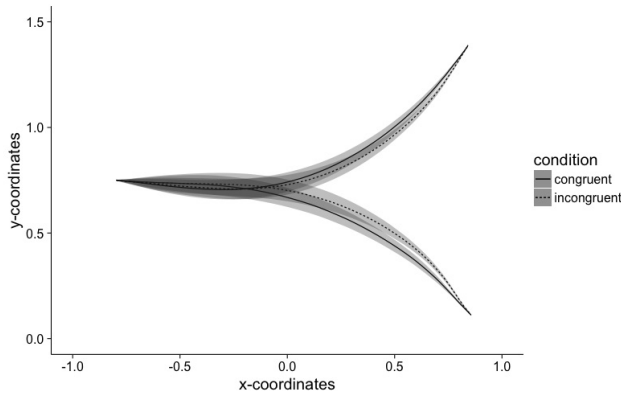


Figure 9. Average computer mouse trajectories in Experiment 2 as a function of spatial congruity condition (congruent versus incongruent) and response side (leftward versus rightward). Shading represents one standard error, computed from the mean  $x$ -coordinates of trajectories over the sample of 32 participants.

symmetric trajectories would be mirrored about the horizontal line defined by the equation  $y = 0.75$  (since the  $y$  coordinates range from 0 to 1.5). To account for this, I computed the asymmetry coefficient  $\mathcal{A}$  using the following equation:

$$\mathcal{A} = \frac{1}{101} \left[ \sum_{i=1}^{101} \frac{y_C(i) + y_I(i)}{2} \right] - 0.75$$

where  $y_C(i)$  is the mean  $y$ -coordinate at timestep  $i$  across all congruent response trials, and  $y_I(i)$  is the mean  $y$ -coordinate at timestep  $i$  across all incongruent response trials. Notice this time that  $\mathcal{A} > 0$  implies an asymmetry characterized by upward bias and  $\mathcal{A} < 0$  implies a downward bias, whereas  $\mathcal{A} = 0$  implies symmetric response trajectories.

As in Experiment 1, the competition account predicts symmetric trajectories ( $\mathcal{A} = 0$ ) for all conditions, but the di-

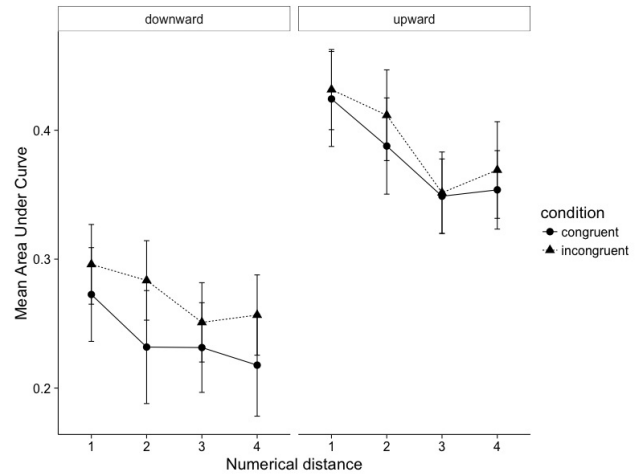


Figure 10. Mean area under the curve (AUC) in Experiment 2 as a function of numerical distance (1, 2, 3, 4), spatial congruity condition (congruent versus incongruent), and response side (leftward versus rightward). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

rect mapping account predicts various asymmetries. Specifically, for small targets, the direct mapping account predicts that as numerical distance increases,  $\mathcal{A}$  decreases from positive to negative. For large targets, the direct mapping account predicts that as distance increases,  $\mathcal{A}$  increases from negative to positive. Again, the critical signature of the direct mapping account would be an interaction between target size and numerical distance.

To test this, I submitted asymmetry scores  $\mathcal{A}$  to a 2 (target size: smaller than 5, larger than 5)  $\times$  2 (numerical distance: small, large) repeated measures analysis of variance. No terms in the ANOVA model were significant (all  $F$  values less than 0.66). As can be seen in Figure 12, there is a

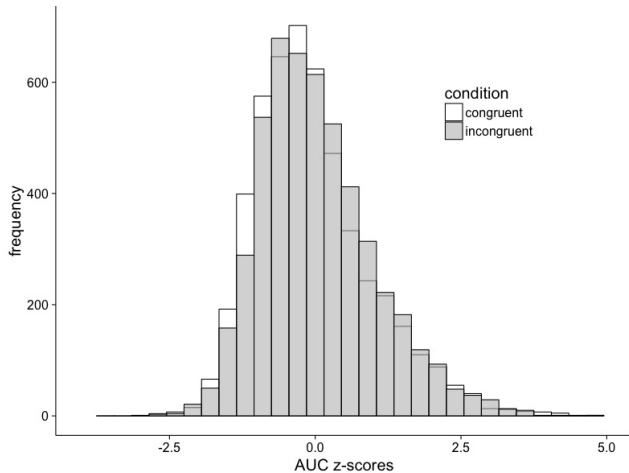


Figure 11. Distribution of z-scores of AUC values in Experiment 2 as a function of spatial congruity condition (congruent versus incongruent).

very slight negative asymmetry overall (reflecting a general bias toward downward mouse movements), but the critical interaction between target decision and numerical distance is absent.

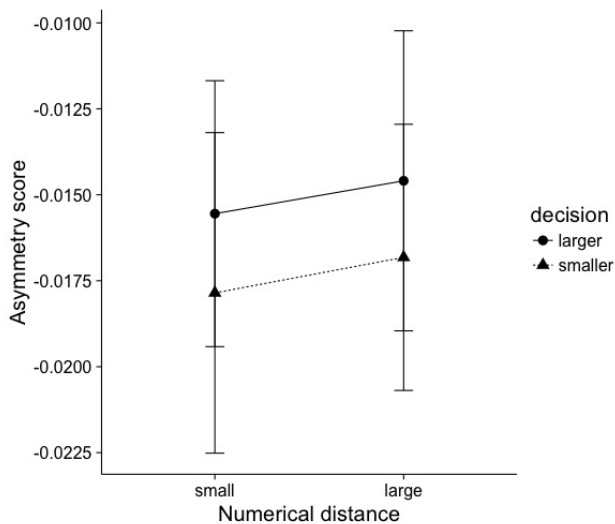


Figure 12. Mean asymmetry score in Experiment 2 as a function of target decision (smaller versus larger) and numerical distance (small versus large). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

## Discussion

In Experiment 2, I replicated the general patterns of results in Experiment 1 using left-to-right mouse movement trajectories instead of the customary bottom-to-top trajec-

ries commonly used in mouse tracking experiments. Specifically, I found that movement trajectories became less curved as numerical distance increased, a result that is only predicted by the competition model (Santens et al., 2011). In addition, I demonstrated that although trajectories exhibited a slight bias toward the bottom of the screen, the patterns of trajectory asymmetries predicted by the direct mapping model (Song & Nakayama, 2008) were absent. In all, these results lend solid support for the competition account of response dynamics in a numerical comparison task.

## General Discussion

The purpose of the present study was to use computer mouse tracking to test between two competing accounts of response dynamics in number comparison. The direct mapping account (Dehaene et al., 1993; Song & Nakayama, 2008) supposes that manual responses correspond directly to a spatial representation of the target number on a mental number line. Alternatively, the competition account (Gevers et al., 2006; Verguts et al., 2005; Santens et al., 2011) posits that curved trajectories result from competition among parallel and partially active response options. Santens et al. (2011) previously tested between these two accounts and found support for the competition model, but they did not test for the possibility that the curved trajectories in the spatially incongruent response condition (LARGER on left, SMALLER on right) could have resulted from averaging across two distinctly different types of responses. In two experiments, I demonstrated that such behaviors did not occur, and the curved trajectories indeed are the result of competitive processing. Furthermore, I performed a novel analysis based on computing an index of asymmetry to show that trajectories for congruent and incongruent trials are essentially mirror-images of each other, which further supports the competition account.

As Santens et al. (2011) explained, a critical test between the direct mapping and competition accounts is discerning the pattern of trajectory curvatures on incongruent trials as numerical distance between the target and comparison standard increases. The direct mapping account predicts that trajectories should become more curved as numerical distance increases (see dashed lines in Figures 1 and 2). The competition account predicts the opposite. In Experiment 1, using area under the curve (AUC) as an index of trajectory curvature, I found that curvature decreased as numerical distance increased, replicating Santens et al. (2011) and supporting the competition account. Additionally, I analyzed the distribution of trajectories (again, as indexed by AUC values) and found that the distribution was not bimodal, which rules out the alternative explanation previously mentioned.

Because the nature of the decreasing pattern of curvatures in Experiment 1 was dependent upon response side (left versus right), I decided to supplement the previous analysis with

a new analysis based on the symmetry of response trajectories across congruent and incongruent trials. As is evident from Figures 1 and 2, one difference between the direct mapping and competition accounts is that while the competition account predicts that congruent and incongruent trials should be symmetric with respect to the movement direction, the direct mapping account would predict directional biases that depend on target size and numerical distance. Is it possible that these biases are being somehow obscured in our present data analyses? To answer this question, I computed an index of asymmetry for each participant that would reflect, on average, whether trajectories are biased leftward or rightward. Specifically, the direct mapping account would predict that for this measure of asymmetry, there would be a significant interaction between target decision (smaller than 5 versus larger than 5) and numerical distance (small versus large). Surprisingly, I found exactly this result; there was a small, but statistically significant interaction between target decision and numerical distance. This pervasive direct mapping signature prevents clean interpretation of the results of Experiment 1.

In Experiment 2, I reasoned that the obtained asymmetry could be the result of biomechanical constraints from the experimental setup. Since every participant used their right hand throughout the task, such a setup could introduce an implicit biomechanical bias. To control for this, I changed the direction of movement from bottom-to-top (the traditional computer mouse tracking direction) to left-to-right. To my knowledge, this is the first experiment in the context of numerical cognition which has used this response direction. The results of Experiment 2 mirrored those of Experiment 1. Critically, there were no asymmetry signatures in Experiment 2, so it is likely the case that the asymmetry found in Experiment 1 was due to biomechanical factors and was not indicative of a direct mapping between the hand and a mental number line. In all, both experiments lend solid support to the competition account of response dynamics in number comparison (Verguts et al., 2005; Santens et al., 2011).

While the present study provides an initial answer to the question of how trajectories evolve in a number comparison task, there are still unsolved problems on the dynamics of the interference effects that arise from numerical distance. Classically, the numerical distance effect has been thought to arise from representational overlap on a mental number line (Dehaene, 1992; Gallistel & Gelman, 1992). Such a model would indicate that the slow-down for small distance number pairs originates at an early representational phase. However, recent mouse tracking work with the size congruity effect (Faulkenberry et al., 2016) has shown that interference effects persist into response execution and are not isolated to early, pre-response representational stages (see also Santens & Verguts, 2011; Buc Calderon et al., 2015). Though not critical to the present study, I did show that distance effects

are found in mouse movement times, but not initiation times, so it appears that the RT effects of numerical distance may be carried into the response stage as well. Note that our instructions forced participants to begin mouse movements very early, and the resulting small initiation times are not likely to reflect cognitive processing (Luce, 1986). Also note that one may model the distance effect using the model of Verguts et al. (2005) with a logarithmic representation of the mental number line and still see such distance effects in response execution<sup>1</sup>. What is unclear is whether the origin of the conflict is in these early representation stages or the late, response execution stages. Thus, computer mouse tracking could be a promising technique to study the timecourse of the numerical distance effect, adding to the debate on early versus late interactions in representations of numerical magnitude (e.g., Arend & Henik, 2015; Santens & Verguts, 2011; Sobel, Puri, & Faulkenberry, 2016).

More broadly, the present work aligns with several recent studies that show support for competitive processing in number tasks ranging among numerical parity (Faulkenberry, 2014), fraction comparison (Faulkenberry et al., 2015), comparison of number pairs (Ganor-Stern & Goldman, 2014), and physical size comparison (Faulkenberry et al., 2016). Such work builds upon recent computational models of number representation (Verguts et al., 2005; Gevers et al., 2006) and adds to the body of work showing that visuo-spatial coding alone is insufficient to account for the observed associations between symbolic number and space (Santens & Gevers, 2008; Gevers et al., 2010). Note that the present data do not completely rule out the mental number line as part of our representation of symbolic number. Indeed, it is likely that a hybrid of these models will best account for existing data (van Dijck, Ginsburg, Girelli, & Gevers, 2014). Further, the present study adds to a growing body of work on the dynamics of cognitive processing, representing diverse topics such as stereotype formation (Freeman & Ambady, 2009), language comprehension (Spivey et al., 2005), memory (Abney, McBride, Conte, & Vinson, 2014; Pappas & Goldinger, 2012), and face processing (Freeman & Ambady, 2011; Hehman, Carpinella, Johnson, Leitner, & Freeman, 2014).

In summary, the present data provides support for a competition model of response dynamics in a numerical comparison task. Using computer mouse tracking, I showed that trajectory curvature decreased as numerical distance from target to standard increases. Critically, this pattern appeared regardless of the spatial congruity of response alternatives (Experiments 1 and 2) or mouse movement direction (Experiment 2). Such behavior cannot be adequately explained by a direct mapping account, and instead reflects the role of competitive processing in symbolic number representation.

<sup>1</sup>Thanks to an anonymous reviewer for pointing this out.

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