THE WORKING MEMORY DEMANDS OF SIMPLE FRACTION STRATEGIES

Thomas J. Faulkenberry (Thomas_Faulkenberry@tamu-commerce.edu)
Texas A&M University – Commerce

The present study examined the roles of phonological and visuo-spatial working memory resources in adults' strategies for comparing the sizes of simple fractions. A dual-task experiment with the choice/no-choice method was used to independently analyze the effects of working memory load (phonological or visuo-spatial) on strategy selection and strategy execution in a fraction comparison task. Load effects for both phonological and visuo-spatial working memory were found, although a concurrent visual working memory load impaired the execution of the fraction comparison task more than did a concurrent phonological load. In addition, selective involvement of working memory as a function of strategy type was found. Conceptual strategies were less affected by concurrent working memory load than were procedural strategies.

Working memory, the ability to store and manipulate information in the short term, is one of the basic functions of human cognition. Perhaps not surprisingly, working memory is vital when people are engaged in a wide variety of complex mathematical tasks (DeStefano & LeFevre, 2004). However, the extent to which the storage and rehearsal functions of working memory are employed depends on the nature of the mathematical task and the specific solution strategy that is used (Hecht, 2002; Imbo & Vandierendonck, 2007a, 2007b). In the present research, I examined the role of working memory in both conceptual and procedural fraction comparison strategies.

According to the working memory model of Baddeley and Hitch (1974) (see also Baddeley, 2007), working memory consists of four interdependent subsystems: the central executive, phonological loop, visuo-spatial sketchpad, and episodic buffer. The central executive is a limited capacity system that is responsible for control, monitoring, response selection, updating, sequencing, and planning. The phonological loop and visuo-spatial sketchpad are secondary systems that allow for the storage and rehearsal of phonological and visuo-spatial information, respectively. The episodic buffer is a system that combines the short-term function of working memory with information from long-term memory.

Previous research has indicated that the phonological loop may be used in complex mental arithmetic to store intermediate results, such as partial sums or products (Ashcraft, 1995). Indeed, recent studies investigating specific mental computational strategies have indicated that people exhibit performance decrements (such as slower reaction times or increased error rates) when doing arithmetic with a nonretrieval strategy while simultaneously holding phonological information in working memory (Imbo & Vandierendonck, 2007a, 2007b). Similar effects of concurrent phonological load have been found for complex mental multiplication (Trbovich & LeFevre, 2003). This lends support to the prediction that people may use phonological working memory resources in a fraction comparison task, particularly when engaged in procedural strategies such as cross-multiplication. It is not yet clear whether phonological resources are necessary for fraction comparison with a more holistic, conceptual strategy, such as benchmarking to common fractions.

The role of the visuo-spatial sketchpad in complex mental arithmetic is less clear. To date, significant visuo-spatial load effects have only been found for vertically-presented 2-digit by 1-digit multiplication problems (Trbovich & LeFevre, 2003) and horizontally- and vertically-presented 2-digit subtraction problems (Imbo & LeFevre, in press). In recent neuroimaging work, Ischebeck, Schocke, and Delazer (2009) found increased activity in the intraparietal sulcus (IPS) when adults were engaged in a fraction comparison task. Along with evidence for the role of the IPS in visuo-spatial working memory (Todd & Marois, 2004), it is possible that people may use visuo-spatial working memory resources in fraction comparison, both in procedural strategies (visually keeping track of intermediate computational results on a mental blackboard) and conceptual strategies (relying on visuo-spatial representations of the two fractions).

The current experiment uses a dual-task method combined with the choice/no-choice method (Siegler & Lemaire, 1997) to investigate the roles of phonological and visuo-spatial working memory in procedural and conceptual strategies for fraction comparison. The choice/no-choice method allows independent analysis of strategy selection and strategy efficiency.

Method

Participants

Fifty-nine undergraduate students at Texas A&M University-Commerce participated in the present experiment (42 women and 17 men). The mean age was 24.6 years (age range 18-56). Participants were selected from the subject pool maintained by the Department of Psychology and Special Education. Participants volunteered for the experiment with no prior knowledge of the tasks or goals of the experiment, lowering the possibility of selection bias based on mathematical ability.

Materials

Fraction stimuli. The stimuli consisted of 48 pairs of proper fractions, divided into 4 disjoint sets of 12. Each set of 12 was constructed by crossing the factors of (a) critical fraction (1/2, 1/3, 2/3), (b) position of the critical fraction (left/right), and (c) relative size of the critical fraction (greater/less). Care was taken to make each of the fraction sets as similar as possible with respect to various structural variables of the fraction pairs, such as the numerical distance between the two fractions and the average cross product, as these variables have been found to significantly predict reaction time (Faulkenberry, 2010; Ischebeck, Shocke, & Delazer, 2009).

Phonological load task. Phonological memory load items were constructed as a list of 48 consonant-vowel-consonant (CVC) nonwords. Across participants, the list was accessed so that each participant received a different problem/load item combination. During each trial in the load condition, participants were asked to subvocally rehearse the CVC nonword while completing a fraction comparison trial. Also, a list of 48 probe CVCs was constructed with half being the same as in the original CVC list and the other half differing from those in the original CVC list by exactly one letter. For example, if the CVC presented before the comparison task was NUQ, the probe item would have been either NUQ or NUW. Participants were asked at the end of each trial whether the probe CVC matched exactly the CVC presented at the beginning of the trial.

Visuo-spatial load task. Visual memory load items were constructed as patterns of 4 asterisks arranged in a 5 x 5 square array. Specifically, a list of 48 different 4-asterisk patterns was constructed. Care was taken to make sure that the patterns of asterisks did not resemble anything recognizable, such as a number or a letter that could be remembered by recalling verbal information. Probe items were constructed in a similar manner to the phonological load task, where non-identical probes were constructed by moving exactly one asterisk by one unit, either up, down, left, or right.

Procedure

Each participant was tested individually at a computer equipped with a button box for input. The experiment took approximately 1 hour to complete. Participants were randomly assigned to either the phonological load condition or the visuo-spatial load condition and solved 12 fraction comparison problems in each of the 6 conditions defined by the 2 (Working memory load: no load, load) 3 (Strategy: Choice, Conceptual only, Procedural only) design. In addition, each participant completed 12 trials of the working memory load task alone. The order of the conditions was counterbalanced across participants with the exception that the choice condition always preceded the conceptual/procedural-only conditions.

Each trial began with the word READY shown in the center of the screen and displayed for 1 second. The word READY then flashed on and off twice at 500-msec intervals. At the end of the last 500-msec interval, the trial stimulus appeared and remained active until the participant responded. In the no-load condition, only a fraction pair appeared, after which the participant was asked either (a) Which strategy did you use? (choice condition) or (b) Were you able to successfully use the required strategy? (conceptual/procedural only conditions). The load task trials were identical, except that the fraction pair was preceded by a memory load item (either a CVC or a visual grid) and followed by the corresponding memory probe item.

Results

Four of the fifty-nine participants were removed from further analysis due to having error rates of 50% or above in the no-load/choice condition. Of the remaining 55 participants, 29 were in the visuo-spatial load condition, and 26 were in the phonological load condition. This resulted in a total of 4,620 trials completed. Of these trials, 264 (5.7%) included an error on the fraction comparison task, and 143 (3.1%) included a failure to use the required strategy in one of the no-choice conditions. All data were analyzed using the multivariate general linear model, and unless otherwise noted, all results were considered to be significant at the alpha = 0.05 level.

Strategy Efficiency

To analyze strategy efficiency, only response times and error scores from the conceptual/procedural-only conditions were included. For each participant, median response times were computed from the trials that included both a correct answer on the fraction comparison and a successful execution of the required strategy. In addition, combined error scores were computed for each participant. The combined error score was computed by recording a trial as an error trial if either (a) an arithmetic error was committed on the fraction comparison task or (b) an error was committed on the load task. A 2 x 2 x 2 multivariate analysis of variance was conducted on correct median RT scores and the -transformed combined error scores with working memory load type (visuo-spatial, phonological) as a between-subjects factor, and working memory load (load, no-load) and strategy (conceptual, procedural) as within-subjects factors (see Table 1).

Univariate analyses revealed no significant differences among the reaction time data. Rather, strategy efficiency effects were found in the combined error scores. The scores were higher for participants in the visuo-spatial load condition (11.3%) than in the phonological load condition (6.46%), $F(1, 53) = 5.35$, partial η^2 = □□092. Combined error scores were also higher under load (17.68%) than under no-load (2.76%), $F(1, 53) = 91.46$, partial η^2 = □□633, and they were higher for benchmarking (11.95%) than for cross-multiplication (6.07%), $F(1, 53) = 16.59$, partial η^2 = □□238 .

Two interaction effects were also significant: load load type, $F(2, 52) = 7.98$, partial η^2 = □□235, and load strategy, $F(2, 52) = 4.58$, partial η^2 = □□□50. Again, this was primarily

Table 1

Median Correct Response Times (in msec) and Combined Error Scores (in %) as a Function of Load Type, Load, and Strategy

		No Load		Load	
Strategy	Measure	M	SE	M	SE
	Phonological Load				
Procedural	RT	2,753	206	2,838	226
	Error	.63	1.1	9.19	1.8
Conceptual	RT	3,162	292	3,131	325
	Error	7.64	2.6	12.54	1.8
	Visuo-spatial Load				
Procedural	RT	2,396	195	2,467	214
	Error	.92	1.0	23.92	1.7
Conceptual	RT	2,891	276	2,462	308
	Error	4.47	2.4	27.86	1.6

due to the pattern of combined error scores. Participants suffered a much higher load penalty in their combined error scores when placed under concurrent visuo-spatial load than they did when placed under phonological load, $F(1, 53) = 16.06$, partial η^2 = □□233. Regarding the load strategy interaction, the load effect on procedural strategies was higher than the load effect on conceptual strategies, $F(1, 53) = 4.23$, partial η^2 = □□074.

The strategy load type interaction was not significant, $F(2, 52) = 0.410$, $p = 0.67$, nor was the three-way interaction of load type, load, and strategy, $F(2, 52) = 0.668$, $p = 0.517$. This indicated that strategy type (procedural / conceptual) is not tied to a specific working memory component. Instead, load effects on specific fraction strategies seem to be load-general.

Strategy Choice

To analyze the effects of working memory load on the choice of strategy used in a fraction comparison, a 2 x 2 analysis of variance was conducted on the percentages of each strategy used with working memory load type (visuo-spatial vs. phonological) as a between-subjects factor and load (load vs. no-load) as a within-subjects factor (see Table 2). There were no effects of load or load type on strategy selection (the highest *F* value was 1.81).

Discussion

The present study found that performance on a fraction comparison task depends on the availability of working memory resources. This was expected given the role that working memory plays in most types of mental arithmetic (DeStefano & LeFevre, 2004). Intriguing findings in this study were the critical interactions of Load Load-Type and Load Strategy. Participants under a visuo-spatial load made significantly more errors (relative to the no-load

Table 2

Mean Percentages of Strategy Choice as a Function of Load Type and Load

	No Load		Load	
Strategy	M	SE	M	SE
	Phonological Load			
Procedural	71.3	8.1	78.5	10.8
Conceptual	28.7	8.1	21.5	10.8
	Visuo-spatial Load			
Procedural	69.4	7.2	57.5	9.8
Conceptual	30.6	7.2	42.5	9.8

condition) than did those participants who were under a phonological load. This interaction effect did not depend on strategy type, indicating a significant role for the visuo-spatial sketchpad in both procedural and conceptual strategies for mental fraction comparison. The phonological load effect was not absent, but it was not as large as the visuo-spatial load effect.

Regarding the Load x Strategy interaction, execution of a procedural strategy suffered more under load than did the execution of a conceptual strategy. This is likely due to the multi-step nature of procedural strategies. Multi-step problems use comparatively more working memory resources than do single-step problems (Ashcraft & Kirk, 2001). The Load x Strategy interaction did not depend on the type of working memory load. It is not immediately clear why this is the case. One may speculate that because all fraction stimuli were composed of single-digit numerators and denominators, the critical role for working memory came at the comparison stage for both strategies. This may imply that both strategies critically involve a magnitude judgment that takes place mostly in the visuo-spatial sketchpad, hence leading to the large visuo-spatial load effect.

The results of the present study provide an important contribution to the overall understanding of adults' numerical and mathematical cognition, but they also have implications in mathematics education regarding the cognitive differences between conceptual and procedural strategies. Future research will need to investigate the visual/spatial distinction in the visuo-spatial sketchpad, and the role of the central executive will also need to be addressed. This future work will add to the overall understanding of the role of working memory in mathematical cognition.

References

Ashcraft, M. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition, 1,* 3-34.

Ashcraft, M., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General, 130,* 224-237.

Baddeley, A. D. (2007). *Working Memory, Thought, and Action.* Oxford, UK: Oxford University Press.

Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. Bower (Ed.), *The psychology of learning and motivation (*Vol. 8, pp. 47-90). New York: Academic Press.

DeStefano, D., & LeFevre, J. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16,* 353-386.

Faulkenberry, T. (2010). *The roles of phonological and visuo-spatial working memory in simple fraction strategies* (Unpublished doctoral dissertation). Texas A&M University – Commerce.

Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition, 30,* 447-455.

Imbo, I., & LeFevre, J. (2010). The role of phonological and visual working memory in complex arithmetic for Chinese and Canadian adults. *Memory & Cognition, 38,* 176-185.

Imbo, I., & Vandierendonck, A. (2007a). Do multiplication and division strategies rely on executive and phonological working memory resources? *Memory & Cognition, 35,* 1759-1771.

Imbo, I., & Vandierendonck, A. (2007b). The role of phonological and executive working memory resources in simple arithmetic strategies. *European Journal of Cognitive Psychology, 19,* 910-933.

Ischebeck, A., Schocke, M., & Delazer, M. (2009). The processing and representation of fractions within the brain. *NeuroImage, 47,* 403-413.

Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General, 126,* 71-92.

Todd, J. J., & Marois, R. (2004). Capacity limit of visual short-term memory in human posterior parietal cortex. *Nature, 428,* 751-754.

Trbovich, P. L., & LeFevre, J. (2003). Phonological and visual working memory in mental addition. *Memory & Cognition, 31,* 738-745.