Getting started in Bayesian Statistics in Psychology: A workshop using JASP

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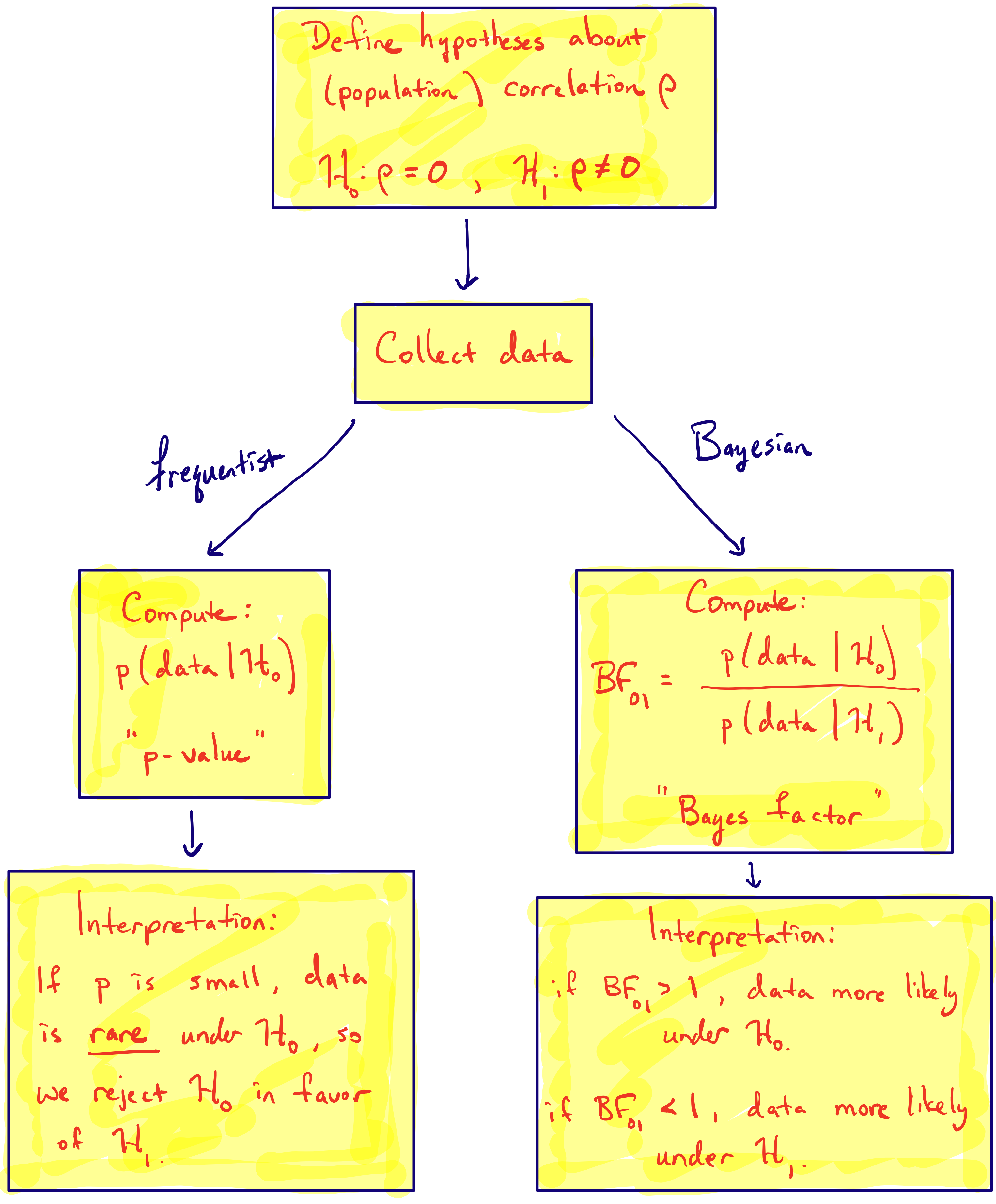
Outline:

- discuss differences between p-values & Bayes factors
- priors on models vs. priors on parameters
- correlation example using JASP, w/ reporting template.
- more resources!

* These slides can be downloaded from https://tomfaulkenberry.github.io/talks.html

Suppose we are interested in the relationship between maths anxiety and performance on a standardized assessment.



p-value = $p(\text{data} \mid \mathcal{H}_0)$

1) only considers fit of $\mathcal{H}_0$ as a potential model for data
2) ignores fit of $\mathcal{H}_1$

Thus, "support" for $\mathcal{H}_1$ is only indirect

Bayes factor = $\dfrac{p(\text{data} \mid \mathcal{H}_0)}{p(\text{data} \mid \mathcal{H}_1)}$

1) considers relative adequacy of both models as predictors of data.
2) can directly index support for either $\mathcal{H}_0$ or $\mathcal{H}_1$.

Ex: $BF_{01} = 8$ → "The observed data are 8 times more likely under $\mathcal{H}_0$ than $\mathcal{H}_1$."

Jeffreys (1961):

BF	Evidence*
1 – 3	anecdotal
3 – 10	moderate
10 – 30	strong
30 – 100	very strong
> 100	extreme

* these are only guidelines!

How does Bayes work?

for single model $\mathcal{H}$:

$$p(\mathcal{H} \mid data) = p(\mathcal{H}) \times \frac{p(data \mid \mathcal{H})}{p(data)}$$

posterior belief in $\mathcal{H}$ = prior belief in $\mathcal{H}$ × updating factor

for <u>two</u> models:

$$\frac{p(\mathcal{H}_0 \mid data)}{p(\mathcal{H}_1 \mid data)} = \frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)} \times \frac{p(data \mid \mathcal{H}_0)}{p(data \mid \mathcal{H}_1)}$$

posterior odds = prior odds × Bayes factor

What do we mean by prior?

Two types of "priors":

1) priors on models

2) priors on parameters within a given model

① Priors on models — before observing data, what is relative likelihood of competing models?

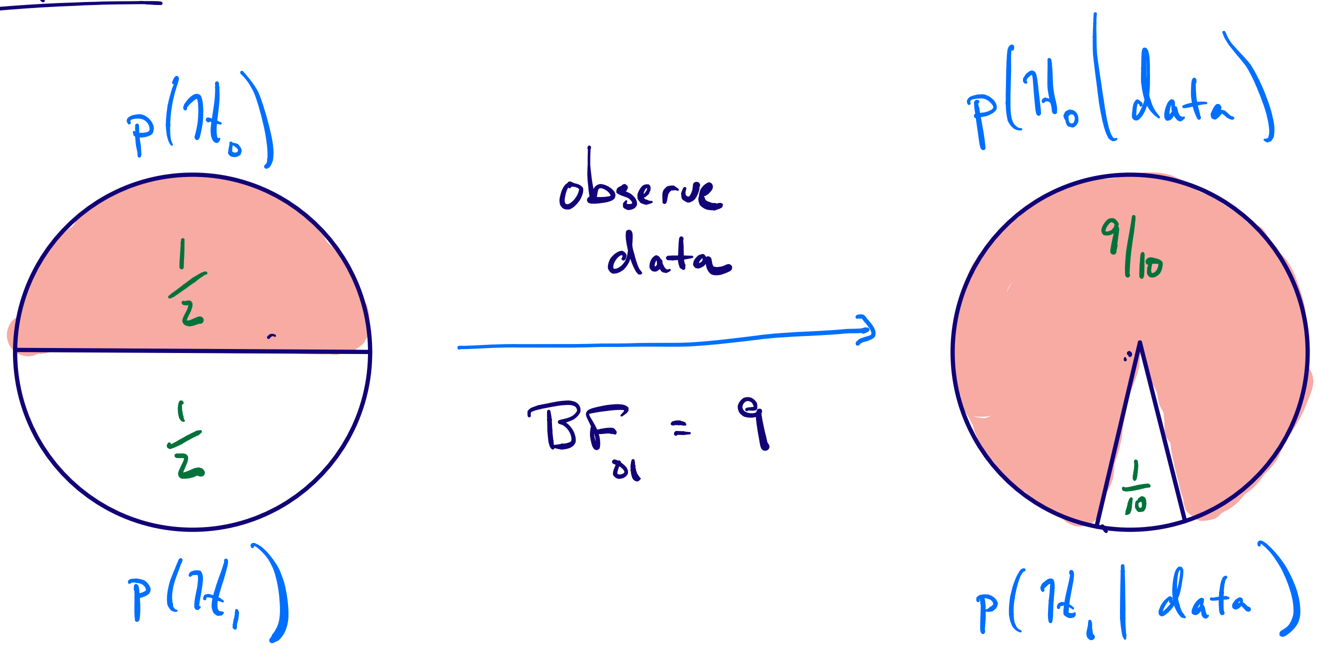
- common default: $p(\mathcal{H}_0) = p(\mathcal{H}_1) = 1/2$
  - ↳ i.e., "1-1 prior odds"
- these prior model probabilities must add to 1
  - ↳ $p(\mathcal{H}_0) + p(\mathcal{H}_1) = \frac{1}{2} + \frac{1}{2} = 1$
- prior model probabilities are updated after observing data:

$$p(\mathcal{H}_0 \mid \text{data}) = \frac{BF_{01} \cdot p(\mathcal{H}_0)}{BF_{01} \cdot p(\mathcal{H}_0) + p(\mathcal{H}_1)}$$

* Note: if $p(\mathcal{H}_0) = p(\mathcal{H}_1) = \frac{1}{2}$,

$$p(\mathcal{H}_0 \mid data) = \frac{BF_{01}}{BF_{01} + 1}$$

Example:



Prior odds = 1 : 1

Posterior odds = 9 : 1

*

$$p(\mathcal{H}_0 \mid data) = \frac{BF_{01}}{BF_{01} + 1} = \frac{9}{9+1} = 0.9$$

② priors on parameters within a given model.

- model definitions : $\mathcal{H}_0 : \rho = 0$

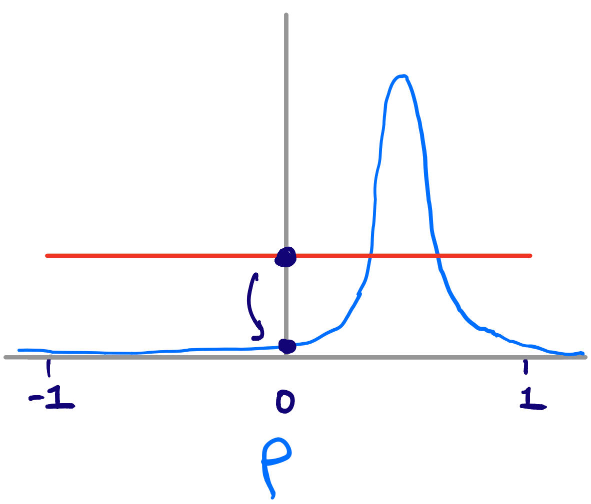
  $\mathcal{H}_1 : \rho \neq 0$ ← what exactly do we mean here?

- we quantify our uncertainty about the correlation ρ under $\mathcal{H}_1$ by placing a distribution on ρ

- suppose we have no idea what to expect. Here, we might believe any value of ρ is equally likely to occur.

  ↳ we say ρ is uniformly distributed on $(-1, 1)$

Prior



"Savage-Dickey" density ratio

→ observe data

Posterior

BF_{01} = factor by which belief that $\rho = 0$ decreases

ρ

$BF_{01} = \frac{1}{10}$

→ $BF_{10} = 10$

Let's continue our working example. Suppose we tested $N = 65$ participants and observed a correlation of $r = 0.37$.

- use JASP "Summary Statistics" module

Elements to report:

1. report results of hypothesis test
   - define $\mathcal{H}_0$, $\mathcal{H}_1$ and specify prior under $\mathcal{H}_1$.

     "Under the null hypothesis we expect a correlation of 0 between maths anxiety and performance. Thus, we define $\mathcal{H}_0 : \rho = 0$. The alternative hypothesis is two-sided, $\mathcal{H}_1 : \rho \neq 0$, and we assigned a uniform prior probability to all values of ρ between -1 and $+1$."

   - report and interpret Bayes factor

     "We found a Bayes factor of $BF_{10} = 13.93$, which means that the observed data are approximately 14 times more likely under $\mathcal{H}_1$ than $\mathcal{H}_0$. This result indicates strong evidence in favor of $\mathcal{H}_1$"

- (optional) calculate and report posterior model probability for preferred model.

  - from earlier,

$$p(\mathcal{H}_1 \mid \text{data}) = \frac{BF_{10}}{BF_{10} + 1}$$

$$= \frac{13.93}{13.93 + 1} = 0.93.$$

- "Assuming prior odds of 1-1 for $\mathcal{H}_1$ and $\mathcal{H}_0$, our observed data updated these odds to 13.93-to-1 in favor of $\mathcal{H}_1$. This is equivalent to a posterior model probability of $p(\mathcal{H}_1 \mid \text{data}) = 0.93$."

2. report results of parameter estimation
   - only if $\mathcal{H}_1$ is the preferred model!
   - specify parameter of interest and remind reader of prior under $\mathcal{H}_1$
     - "of interest is the posterior distribution for ρ, the population-level correlation between maths anxiety and performance. Under $\mathcal{H}_1$, ρ was assigned a uniform prior over the interval from -1 to $+1$."

- report the 95% credible interval.
  - "The posterior distribution for ρ had a median of 0.356, with a central 95% credible interval that ranges from 0.134 to 0.554."

More resources

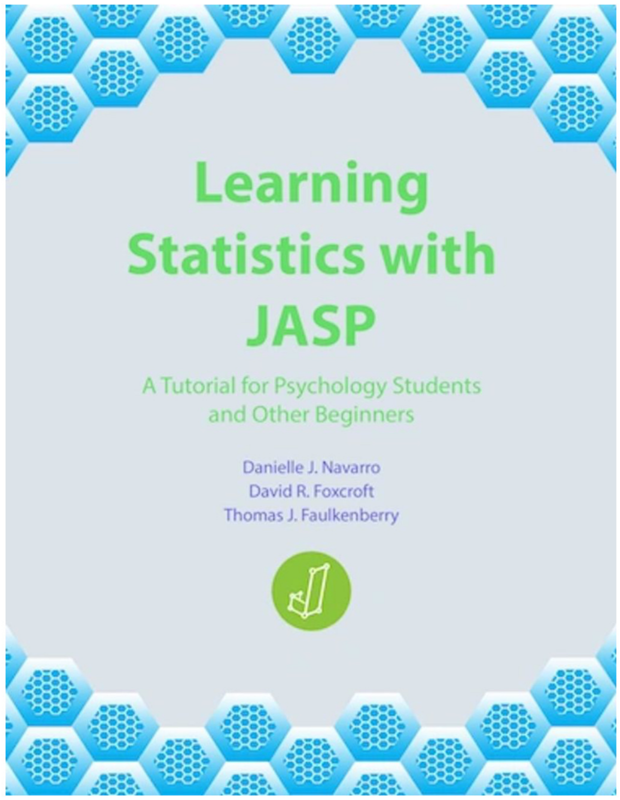
* van Doorn et al. (2021). The JASP guidelines for conducting and reporting a Bayesian analysis. Psychonomic Bulletin & Review

* Faulkenberry, Ly, & Wagenmakers (2020). Bayesian inference in numerical cognition: A tutorial using JASP. Journal of Numerical Cognition

Before you go – a couple of shameless plugs!

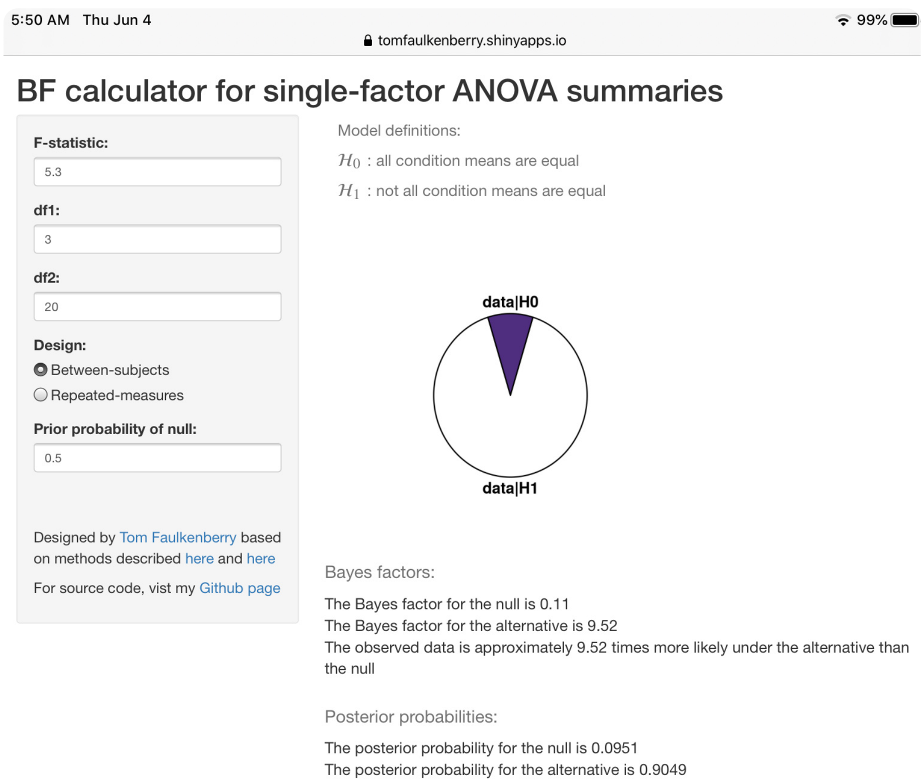
1) our FREE statistics textbook!

www.learnstatswithjasp.com



2) a free online Bayes factor calculator!

https://tomfaulkenberry.shinyapps.io/anovaBFcalc



Take home points:

- Bayes is easy, especially with the right software.
- Bayes answers the questions you thought you were asking
- testing or estimation? No need to choose — Bayes gives you both!

More questions — contact me!

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