A systems factorial technology approach to classifying the architecture of fraction perception

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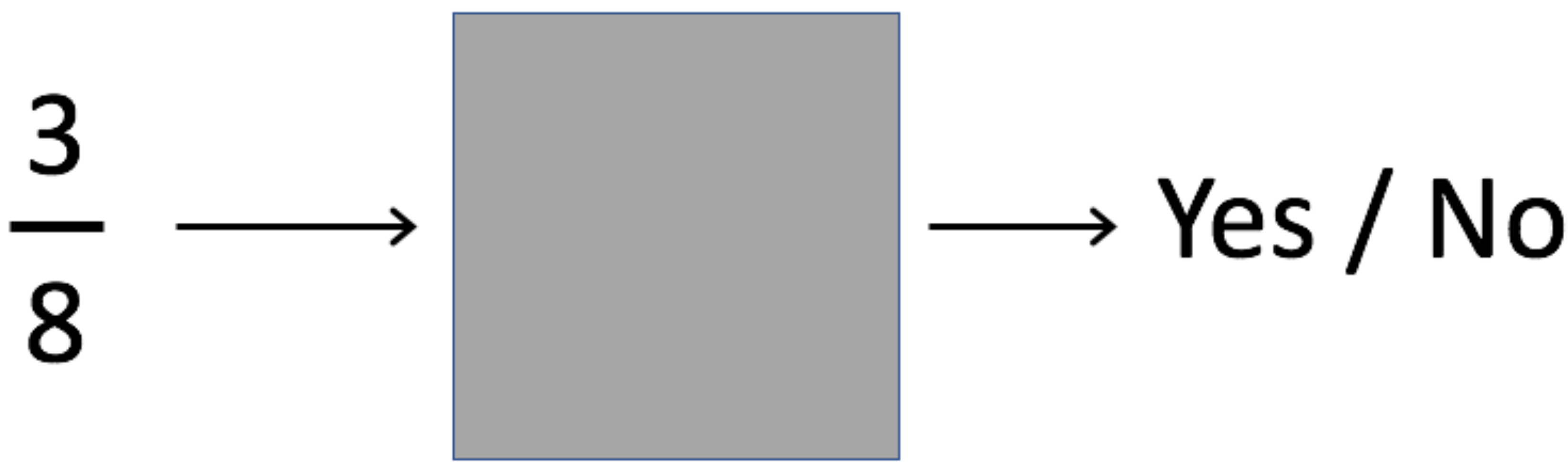
Task: decide if fraction contains a number greater than 5 in *either* component.

Question: how do we make this decision?



Imagine our mental "factory" has two workers, Nelson and Dani, responsible for making the decision for the numerator and denominator, respectively.

Some possibilities:

- Nelson looks at numerator first, then passes the fraction to Dani, who looks at denominator (regardless of Nelson's decision)

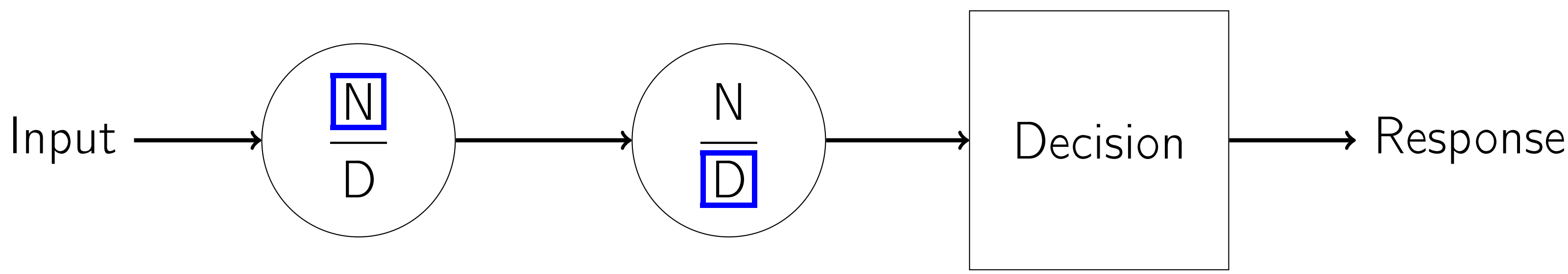
- Nelson looks at numerator first, only passing to Dani if her component doesn't satisfy "greater than 5" condition.

- Nelson and Dani look at their components at the same time, and fraction is passed on once both Nelson and Dani have made their respective decisions

- Nelson and Dani look at their components at the same time. If one of them finds that their component satisfies "greater than 5" condition, the fraction is immediately passed on.

Serial architecture

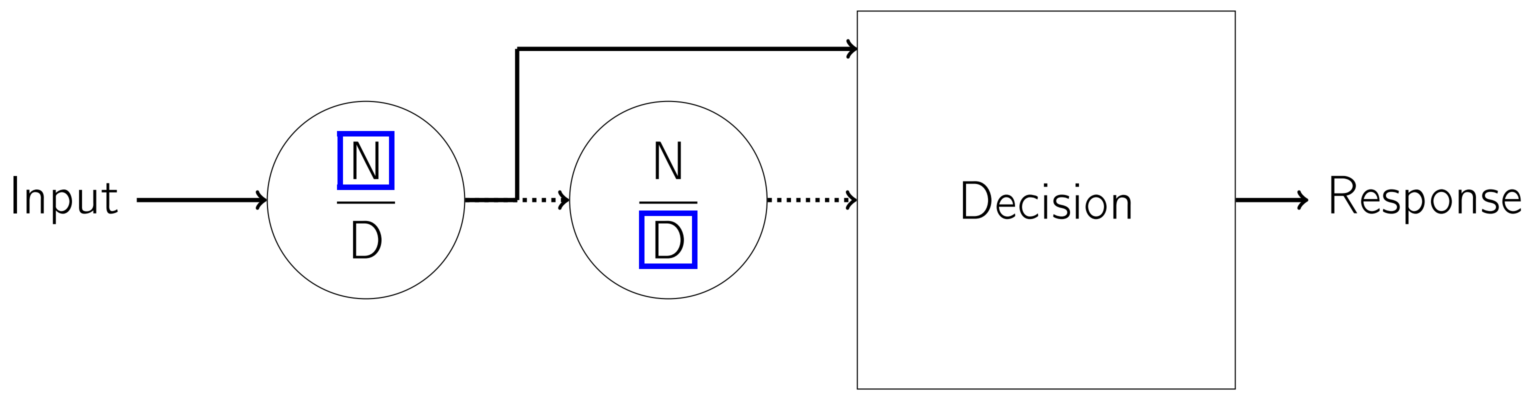
Stopping rule = exhaustive



Each target is processed *sequentially* – both N and D must complete before response is made

Serial architecture

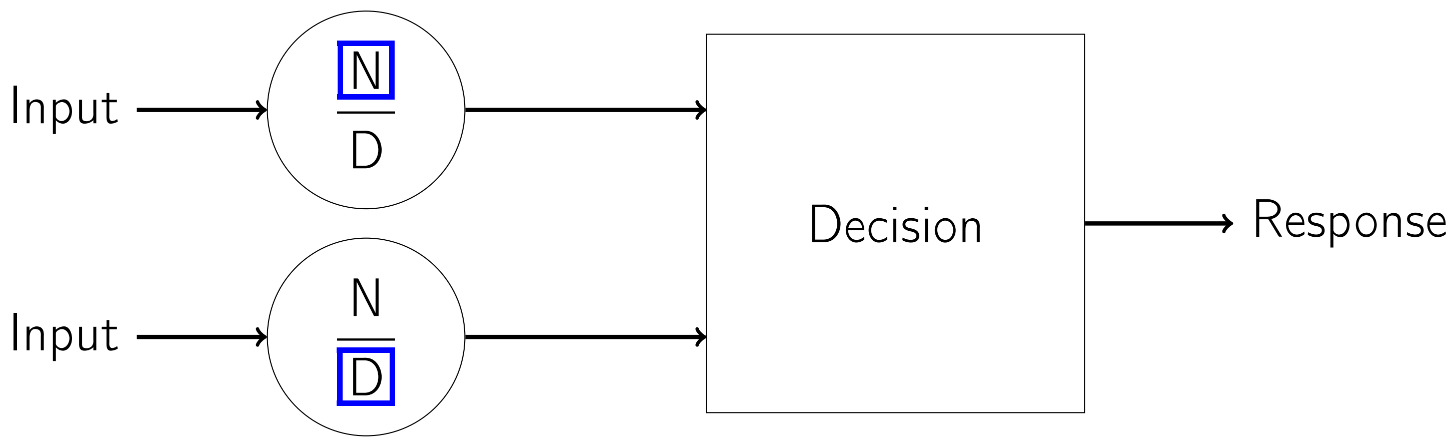
Stopping rule = self-terminating



Each target is processed *sequentially* – but either N or D is sufficient to trigger response

Parallel architecture

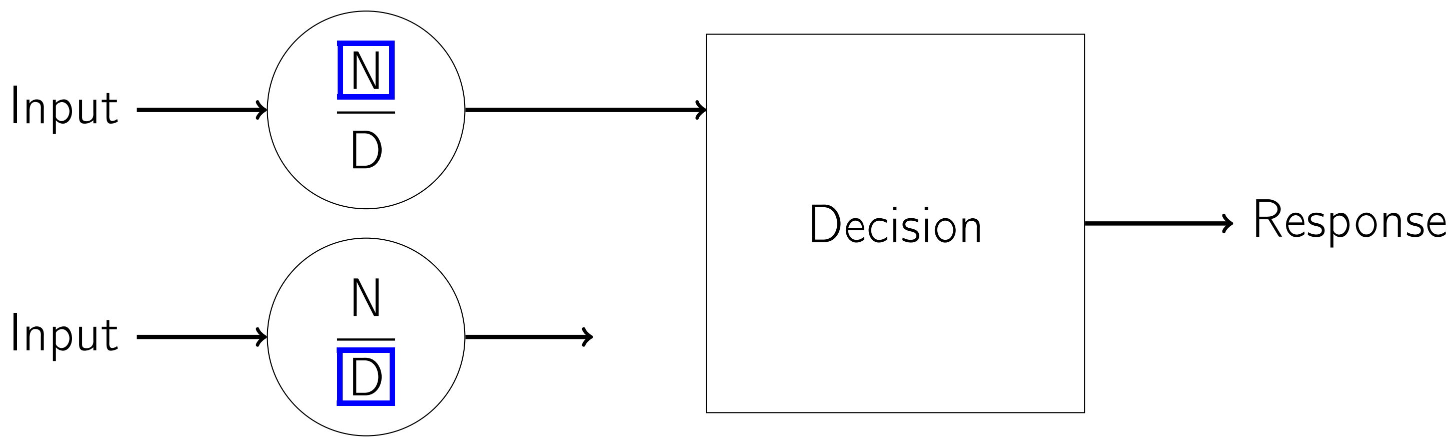
Stopping rule = exhaustive



Each target is processed *simultaneously* – both N and D must complete before response is made

Parallel architecture

Stopping rule = self-terminating



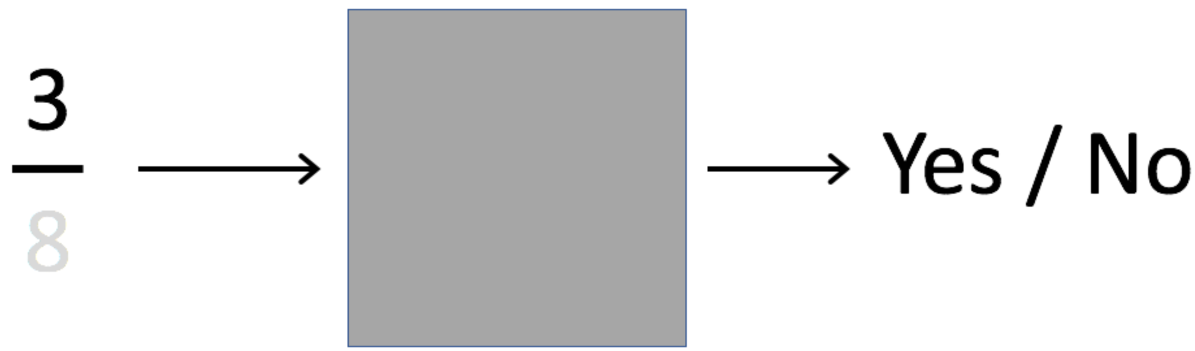
Each target is processed *simultaneously* – but either N or D is sufficient to trigger response

Our goal is to determine which of these architectures governs how we process fractions.

- Parallel exhaustive
- Parallel self-terminating
- Serial exhaustive
- Serial self-terminating

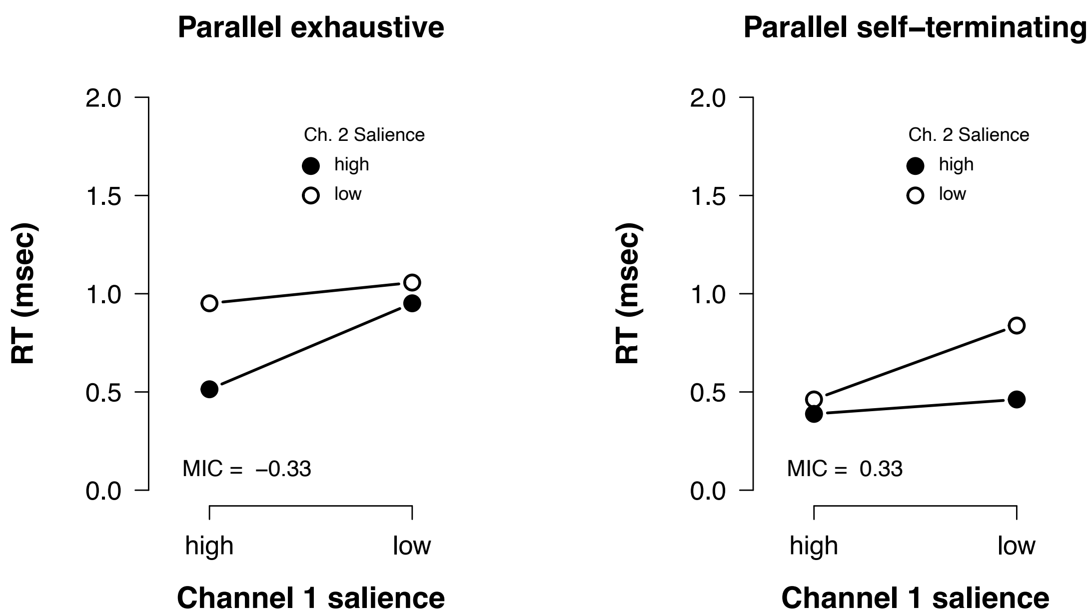
Unfortunately, we cannot *directly* observe how our "workers" Nelson and Dani handle their respective tasks.

However, we can *indirectly* observe them by manipulating the inputs they receive and measuring the effect on performance.

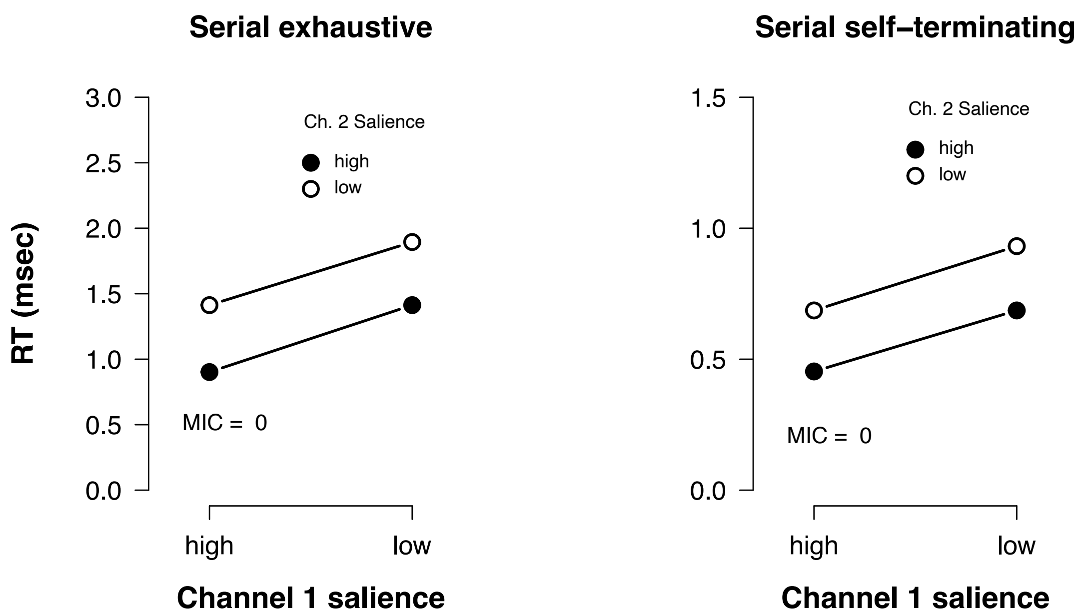


We call this a **salience** manipulation. The goal is to make the task harder by manipulating how easy it is for Nelson/Dani to make their decisions.

Classically (e.g., Sternberg), problems of this type have been studied by comparing **mean** response times (RTs) in each of the salience conditions.



Classically (e.g., Sternberg), problems of this type have been studied by comparing **mean** response times (RTs) in each of the salience conditions.



Problem – model mimicry: these techniques cannot distinguish between different stopping rules for serial architecture.

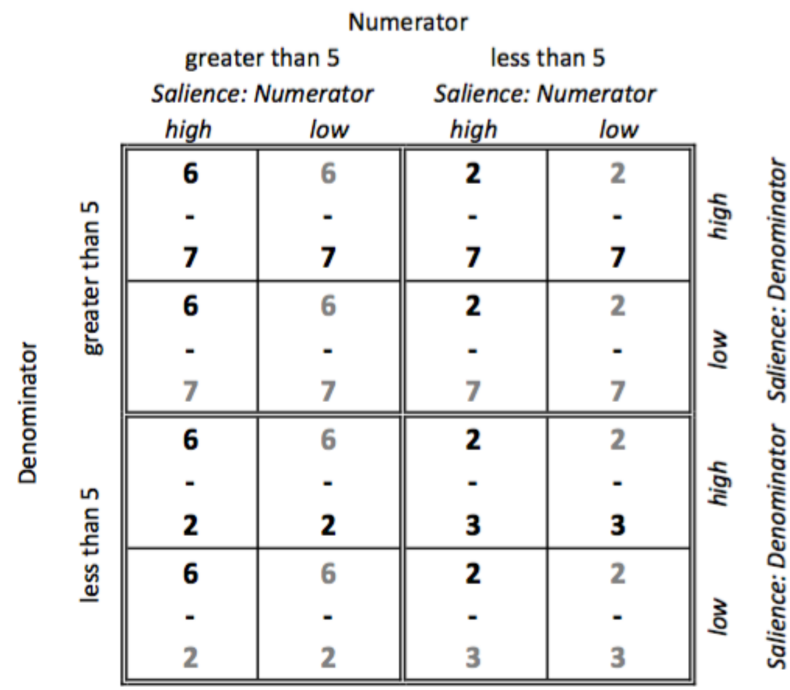
Solution: model the entire *distribution* of RTs and use tools of *systems factorial technology* (Townsend & Nozawa, 1995; Houpt, Heathcote, & Eidels, 2017)

- Consider system as two processing channels with completion times T_N and T_D (not observed)
- build 4 models of RT, the total completion time (observed)
  - $\mathcal{M}_1$: parallel exhaustive $\rightarrow RT = \max(T_N, T_D)$
  - $\mathcal{M}_2$: parallel self-terminating $\rightarrow RT = \min(T_N, T_D)$
  - $\mathcal{M}_3$: serial exhaustive $\rightarrow RT = T_N + T_D$
  - $\mathcal{M}_4$: serial self-terminating $\rightarrow RT = \begin{cases} T_N & \text{with probability } p \\ T_D & \text{with probability } 1-p \end{cases}$

Task: decide if fraction contains a number greater than 5 in *either* component.

Stimuli:

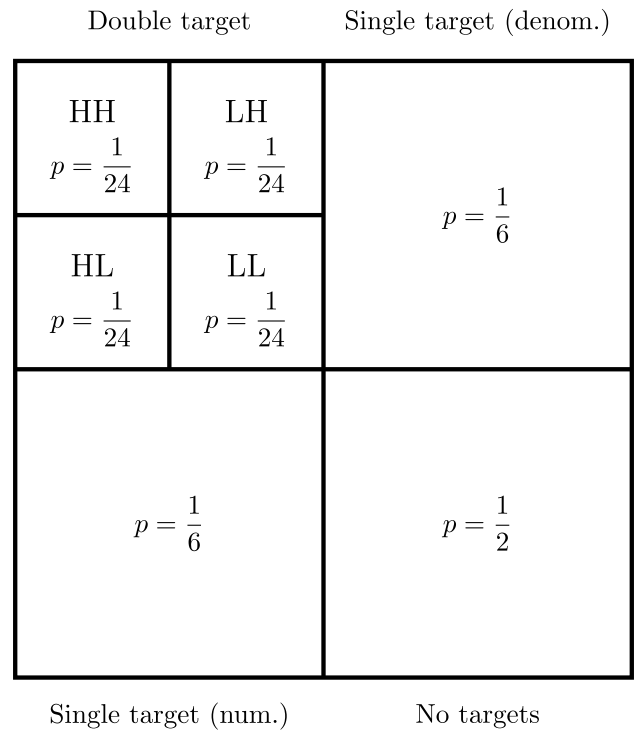
- numerators = 2,3,4,6,7,8
- denominators = 2,3,4,6,7,8
- 36 possible fractions
- how many times do we repeat them?



Task: decide if fraction contains a number greater than 5 in *either* component.

Stimuli:

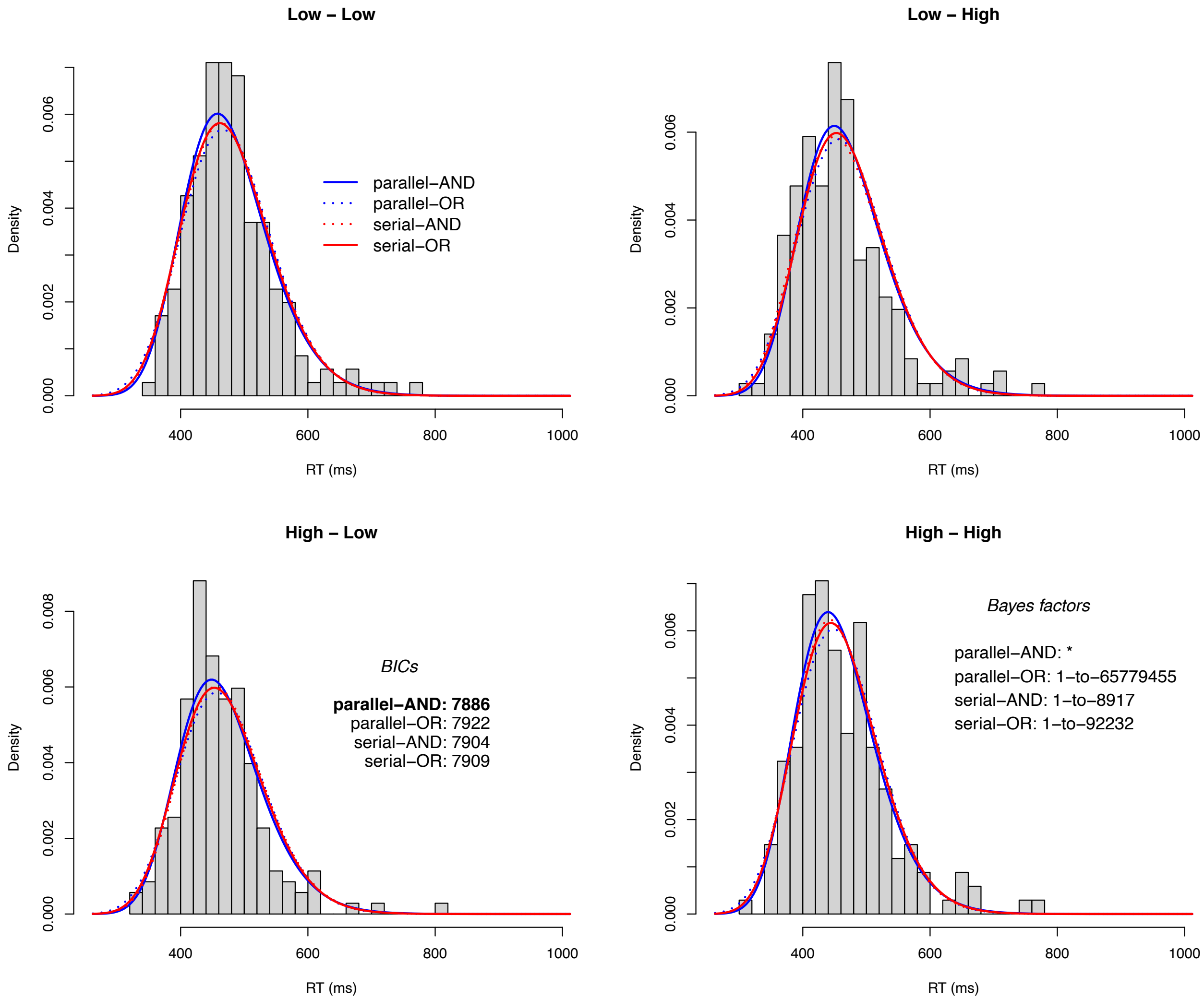
- 36 fractions
- need between 100 and 200 trials in each double target condition
- 5 reps of 36 = 180 trials
- 180 trials = 1/24 of stimulus set
- $24 \times 180 = 4,320$ trials



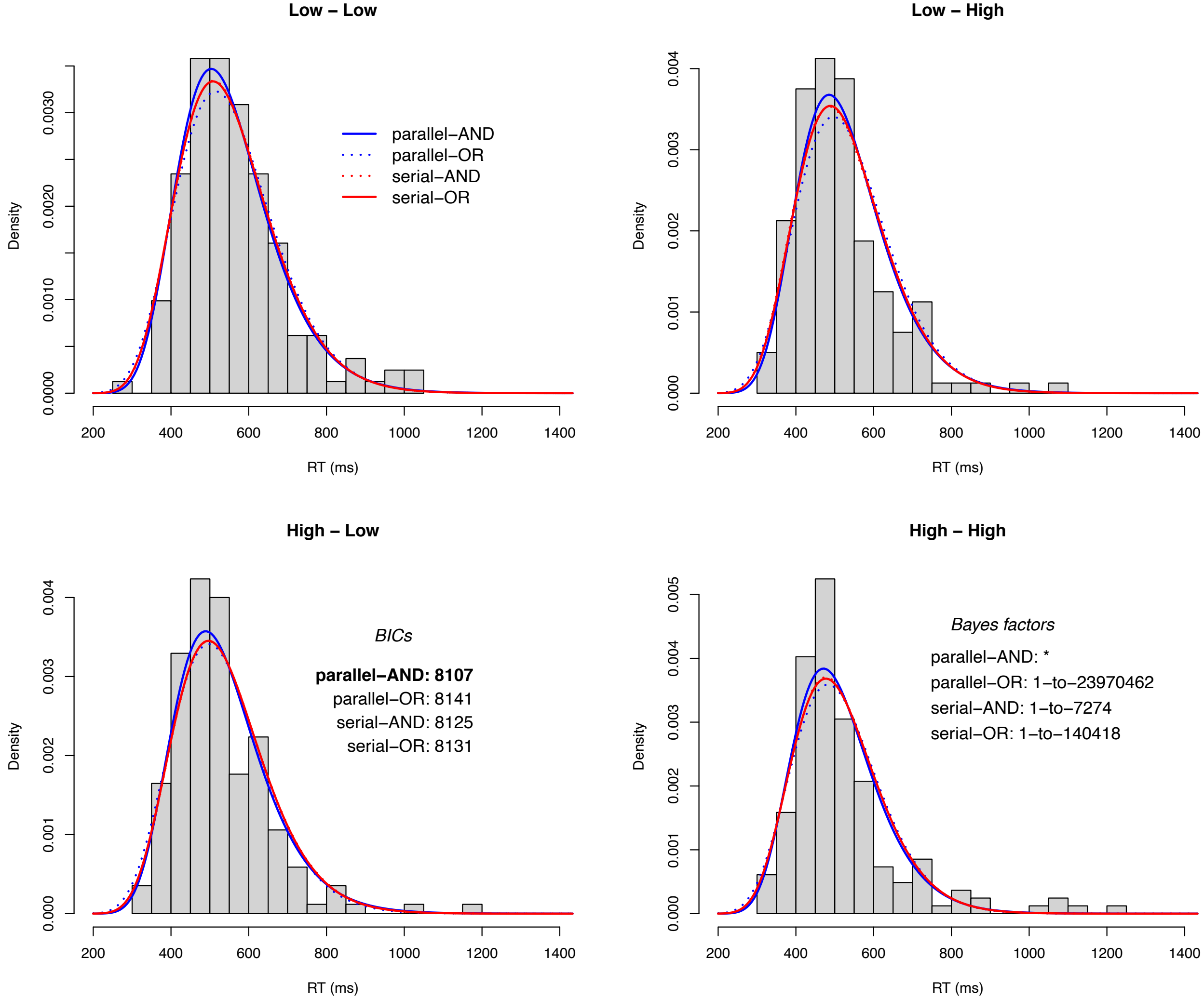
Modeling workflow: for each of our $N = 5$ observers, we:

- filter out errors ($M = 3.75\%$) and RT outliers ($M = 1.6\%$)
- fit each model to observed RTs via maximum likelihood estimation
- index model fit by BIC
  - $\text{BIC} = k \log(N) - 2 \log\Big(p(y \mid \theta_{\text{max}})\Big)$
  - smaller BIC = better model fit
- estimate predictive adequacy of each model with Bayes factor
  - $\text{BF}_{12} \approx \exp\left(\frac{\text{BIC}_2 - \text{BIC}_1}{2}\right)$

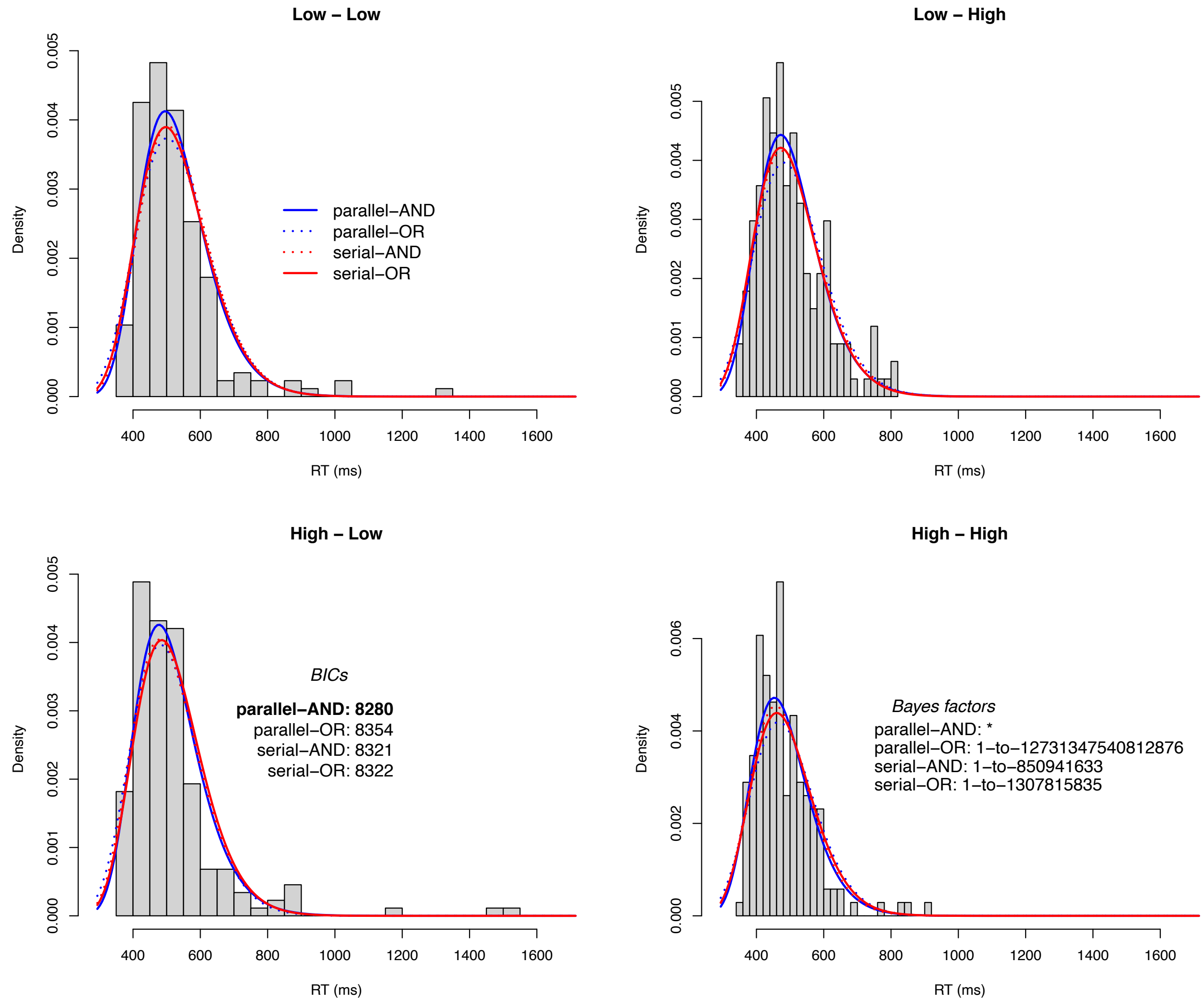
Observer 1:


Low – Low
Low – High
High – Low
High – High
Density
RT (ms)
parallel–AND
parallel–OR
serial–AND
serial–OR
BICs
parallel–AND: 7886
parallel–OR: 7922
serial–AND: 7904
serial–OR: 7909
Bayes factors
parallel–AND: *
parallel–OR: 1–to–65779455
serial–AND: 1–to–8917
serial–OR: 1–to–92232

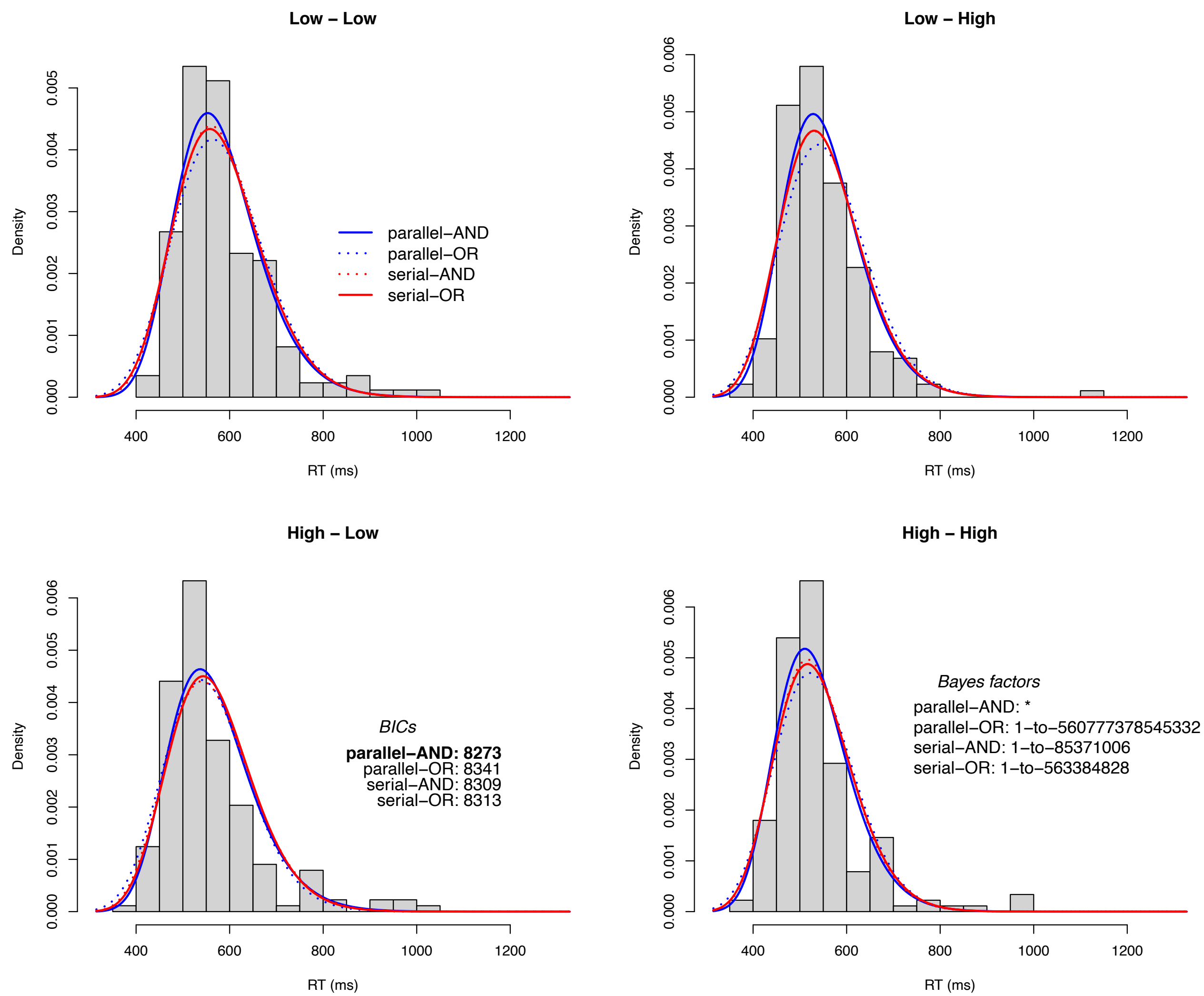
Observer 2:


Low – Low
Low – High
High – Low
High – High
parallel–AND
parallel–OR
serial–AND
serial–OR
BICs
parallel–AND: 8107
parallel–OR: 8141
serial–AND: 8125
serial–OR: 8131
Bayes factors
parallel–AND: *
parallel–OR: 1–to–23970462
serial–AND: 1–to–7274
serial–OR: 1–to–140418
Density
RT (ms)

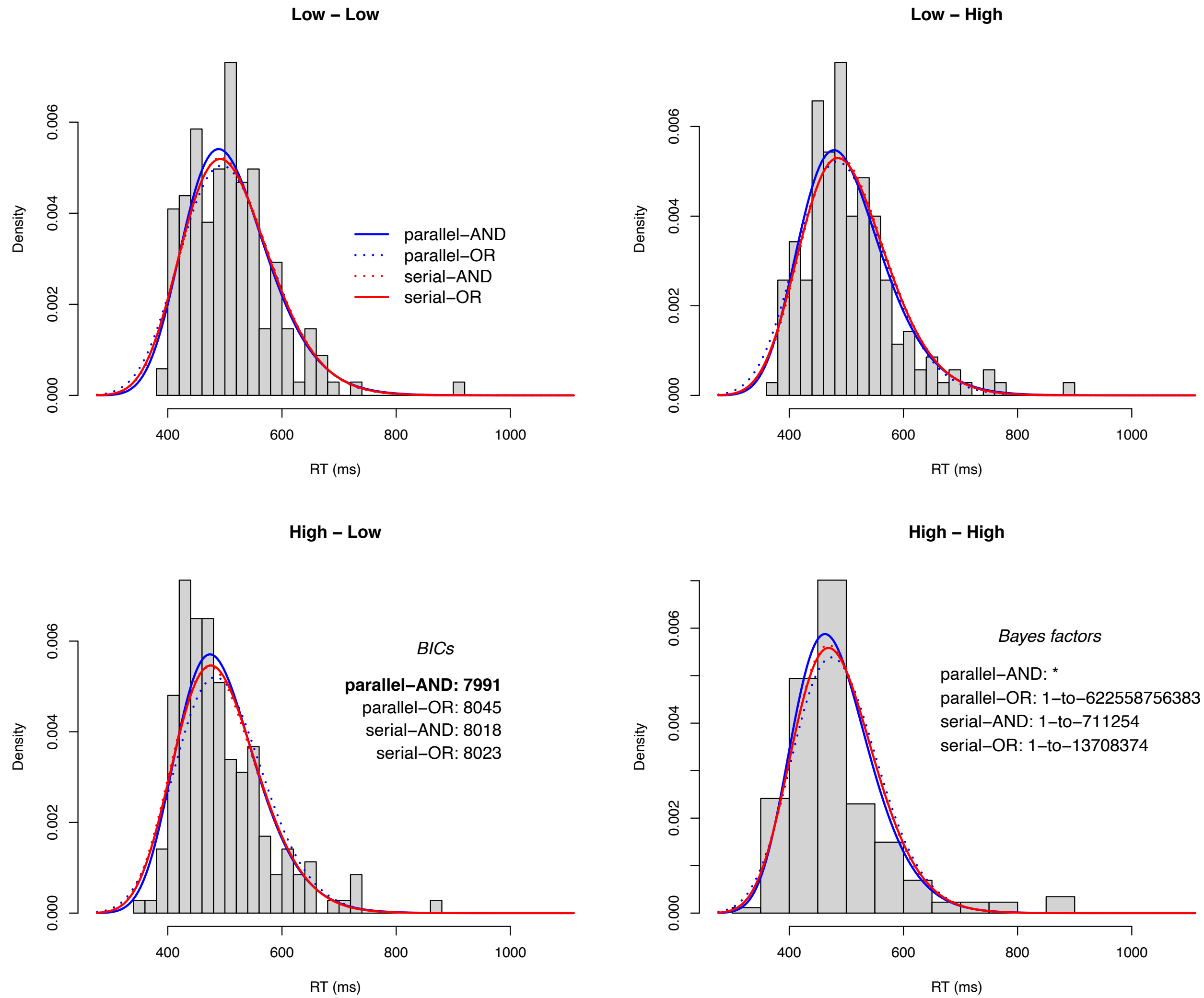
Observer 3:


Low – Low
Low – High
High – Low
High – High
Density
RT (ms)
parallel–AND
parallel–OR
serial–AND
serial–OR
BICs
parallel–AND: 8280
parallel–OR: 8354
serial–AND: 8321
serial–OR: 8322
Bayes factors
parallel–AND: *
parallel–OR: 1–to–12731347540812876
serial–AND: 1–to–850941633
serial–OR: 1–to–1307815835

Observer 4:


Low – Low
Low – High
High – Low
High – High
Density
RT (ms)
parallel–AND
parallel–OR
serial–AND
serial–OR
BICs
parallel–AND: 8273
parallel–OR: 8341
serial–AND: 8309
serial–OR: 8313
Bayes factors
parallel–AND: *
parallel–OR: 1–to–560777378545332
serial–AND: 1–to–85371006
serial–OR: 1–to–563384828

Observer 5:



Low – Low
Low – High
High – Low
High – High
Density
RT (ms)
parallel–AND
parallel–OR
serial–AND
serial–OR
BICs
parallel–AND: 7991
parallel–OR: 8045
serial–AND: 8018
serial–OR: 8023
Bayes factors
parallel–AND: *
parallel–OR: 1–to–622558756383
serial–AND: 1–to–711254
serial–OR: 1–to–13708374

Interim thoughts:

- Fraction components seem to be processed in a *parallel exhaustive* manner
- Next questions:
  - how might model fits be improved?
  - do conclusions change with different model specifications?
  - can we improve the salience manipulation (i.e., better separation of RT distributions)?
  - how can this method be applied to other fundamental questions in numerical cognition?

Thanks!

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