Testing the independence of encoding and calculation in mental addition: A confirmatory Bayesian analysis

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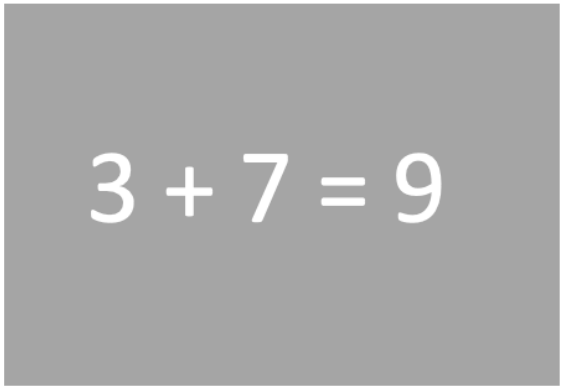
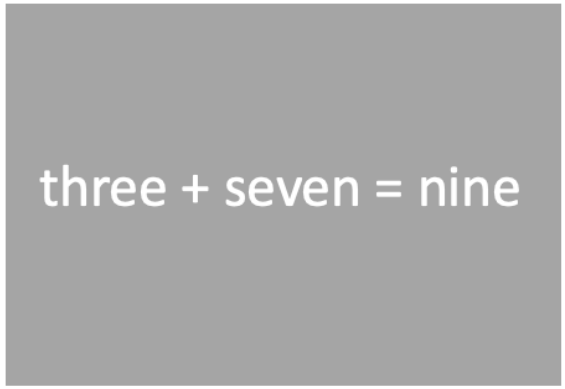
How does surface format impact the solution of arithmetic problems?



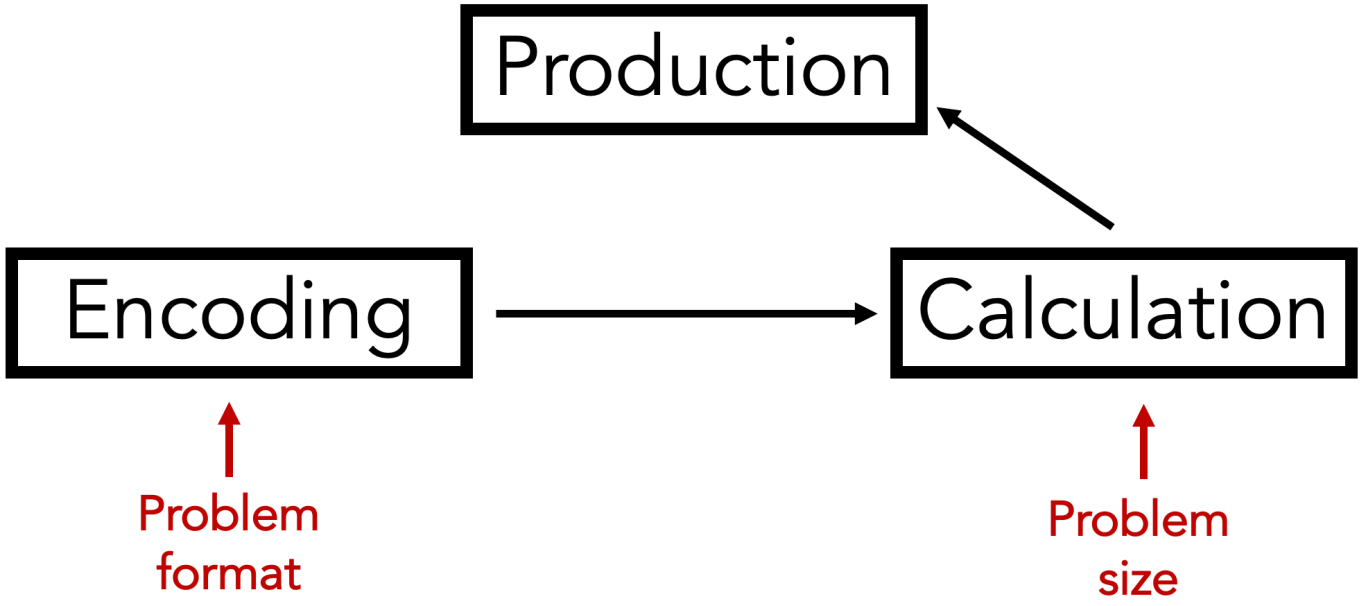


- are effects localized to encoding? (e.g., Dehaene & Cohen, 1995; McCloskey, 1992; Noël, Fias, & Brysbaert, 1997)

- do effects penetrate downstream to directly affect calculation? (e.g., Blankenberger & Vorberg, 1997; Campbell, 1994; Campbell & Fugelsang, 2001)

Two competing accounts:

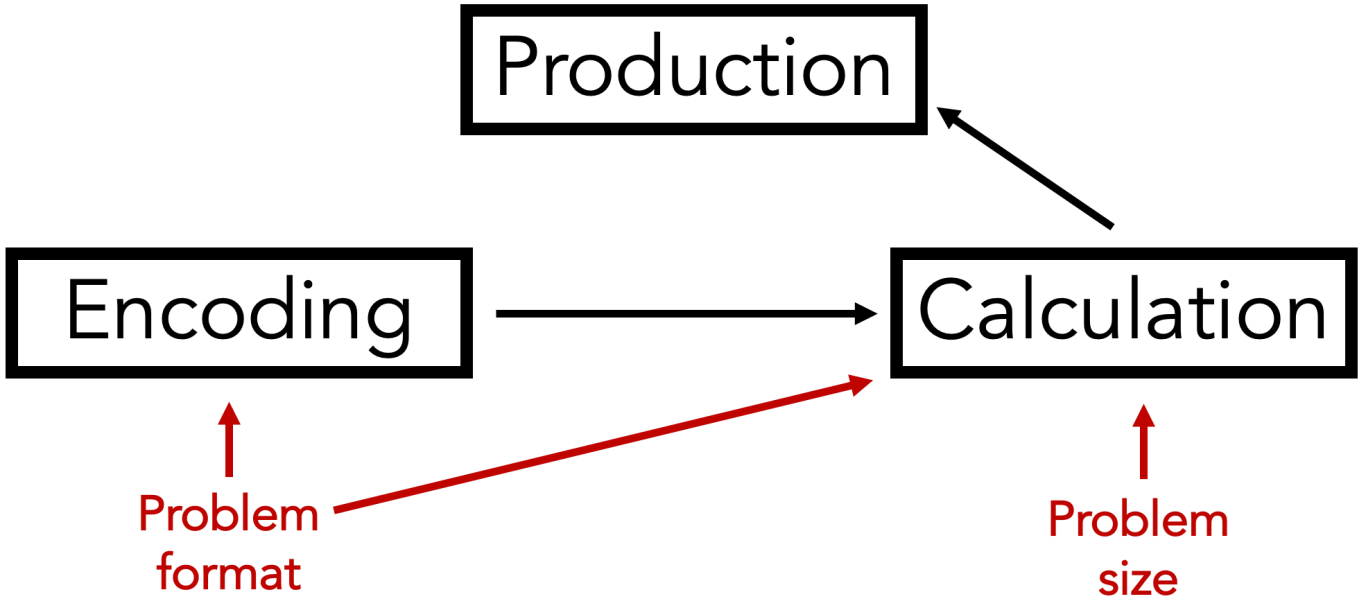
1. Additive model



$\mathcal{M}_1$: RT $\sim$ format + size

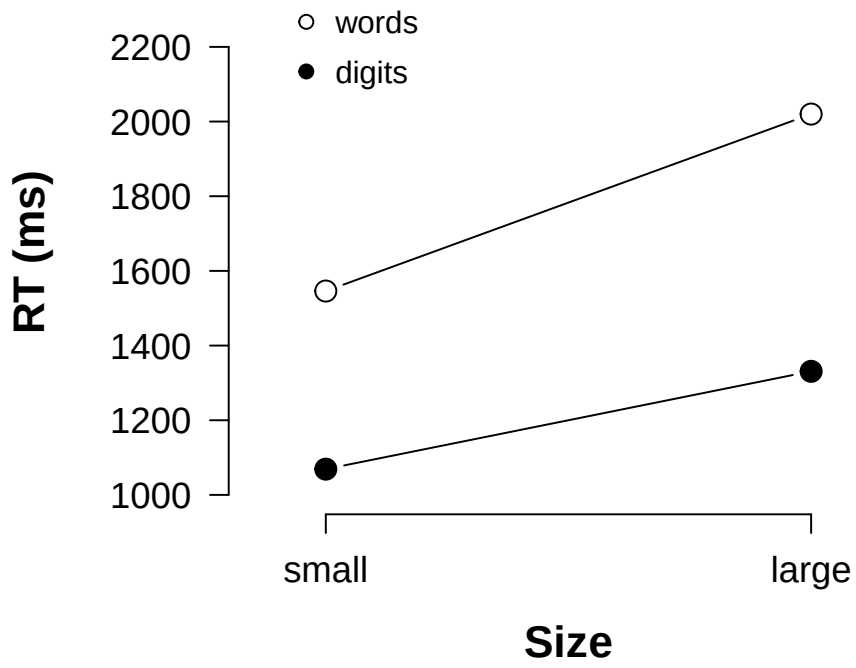
Two competing accounts:

2. Interactive model



$\mathcal{M}_2$: RT $\sim$ format + size + format · size

Campbell & Fugelsang (2001) - evidence for interactive account in addition verification



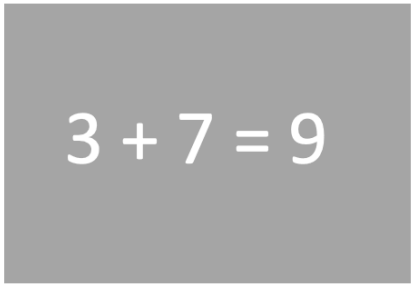
Format effect larger for large problems (+689 ms) than for small problems (+477 ms)

Present work – large-scale replication of Campbell & Fugelsang (2001)

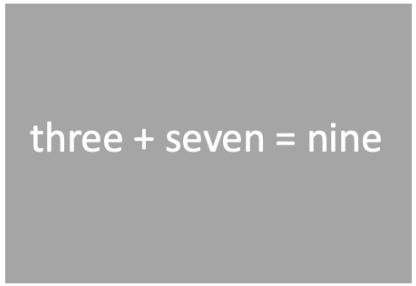
1000 msec

Participants: *N* = 233

Design: 2 (Problem size: Small vs. Large)
x 2 (Format: Digits vs. Words)
x 2 (Truth value: True vs. False)



or



button press
TRUE / FALSE

Analytic workflow:

1. Remove errors (7.5%) and long trials (0.7%)

2. Separate RTs into $2 \times 2 \times 233$ design cells

3. Collapse design cells (median)

4. Submit medians to Bayesian RM ANOVA in JASP

   (a) random intercepts only (Rouder et al., 2012)
   (b) random intercepts and slopes (van den Bergh et al., 2022)

Which RM ANOVA do I use?

- Older JASP versions ($\leq 0.16.2$) specify random intercepts only

$$
\begin{aligned}
Y_{si} &= \beta_0 + S_{0s} + \beta_1 X_1 + \varepsilon_{si} \ , \\
S_{0s} &\sim \mathcal{N}(0, \tau_{00}^2) \ , \\
\varepsilon &\sim \mathcal{N}(0, \sigma^2) \ .
\end{aligned}
$$

This model specification assumes no subject-level variability in the main effects.

Which RM ANOVA do I use?

- Newer JASP ($\geq 0.16.3$) versions includes random slopes on the main effects.

$$
\begin{aligned}
Y_{si} &= \beta_0 + S_{0s} + (\beta_1 + S_{1s})X_1 + \varepsilon_{si} \ , \\
(S_{0s}, S_{1s}) &\sim \mathcal{N}\left(0, \begin{bmatrix} \tau_{00}^2 & \rho\tau_{00}\tau_{11} \\ \rho\tau_{00}\tau_{11} & \tau_{11}^2 \end{bmatrix}\right) \ , \\
\varepsilon &\sim \mathcal{N}(0, \sigma^2) \ .
\end{aligned}
$$

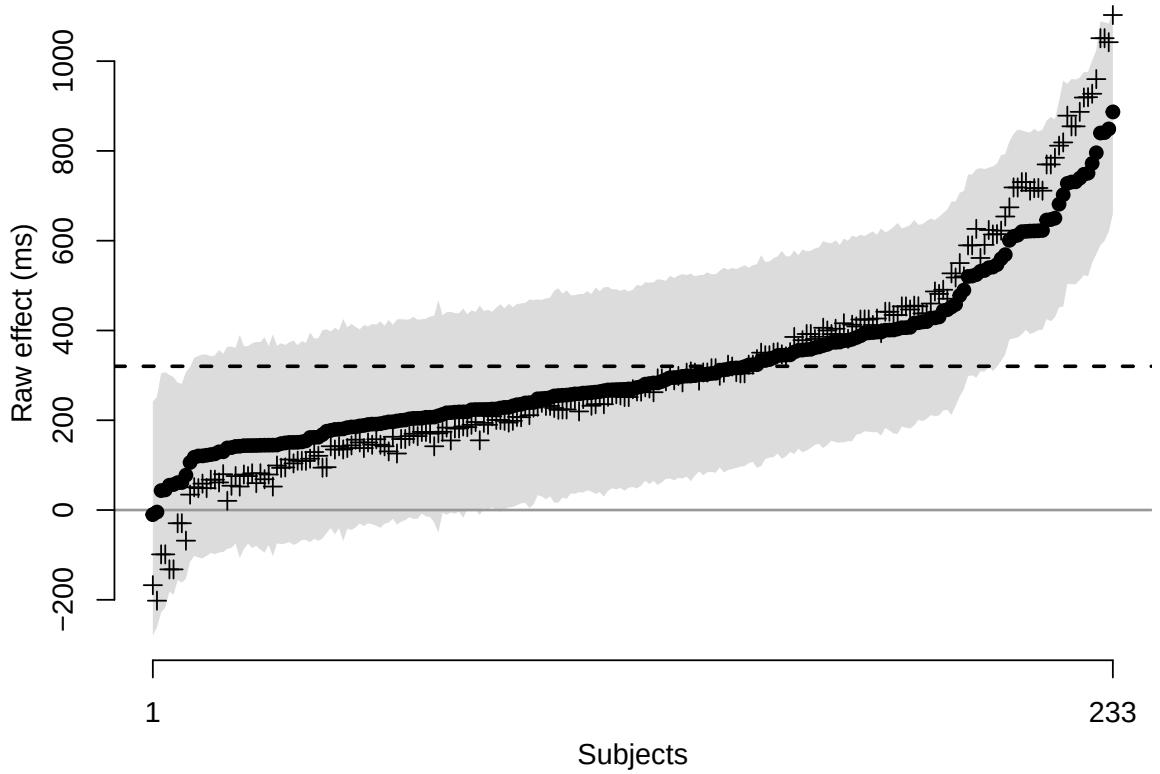
This model specification adds subject-level variability in the main effects.

Which RM ANOVA do I use?

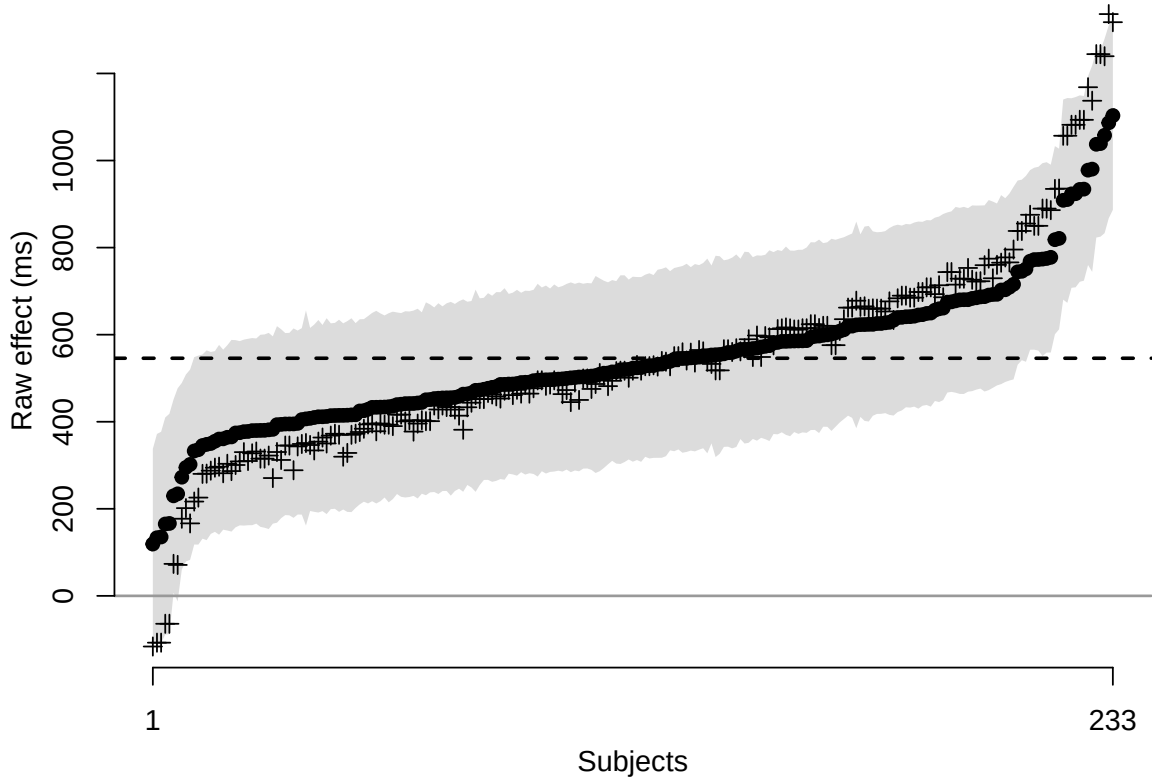
- Is it reasonable to assume that there are individual differences in the main effects?

- What is the structure of individual differences in size effect? Format effect?
  - use method of Haaf & Rouder (2017) to build hierarchical Bayesian model of individual differences in each of the main effects.
  - see also Faulkenberry & Bowman (in press, J. of Cog. Psy.)

Size effect:



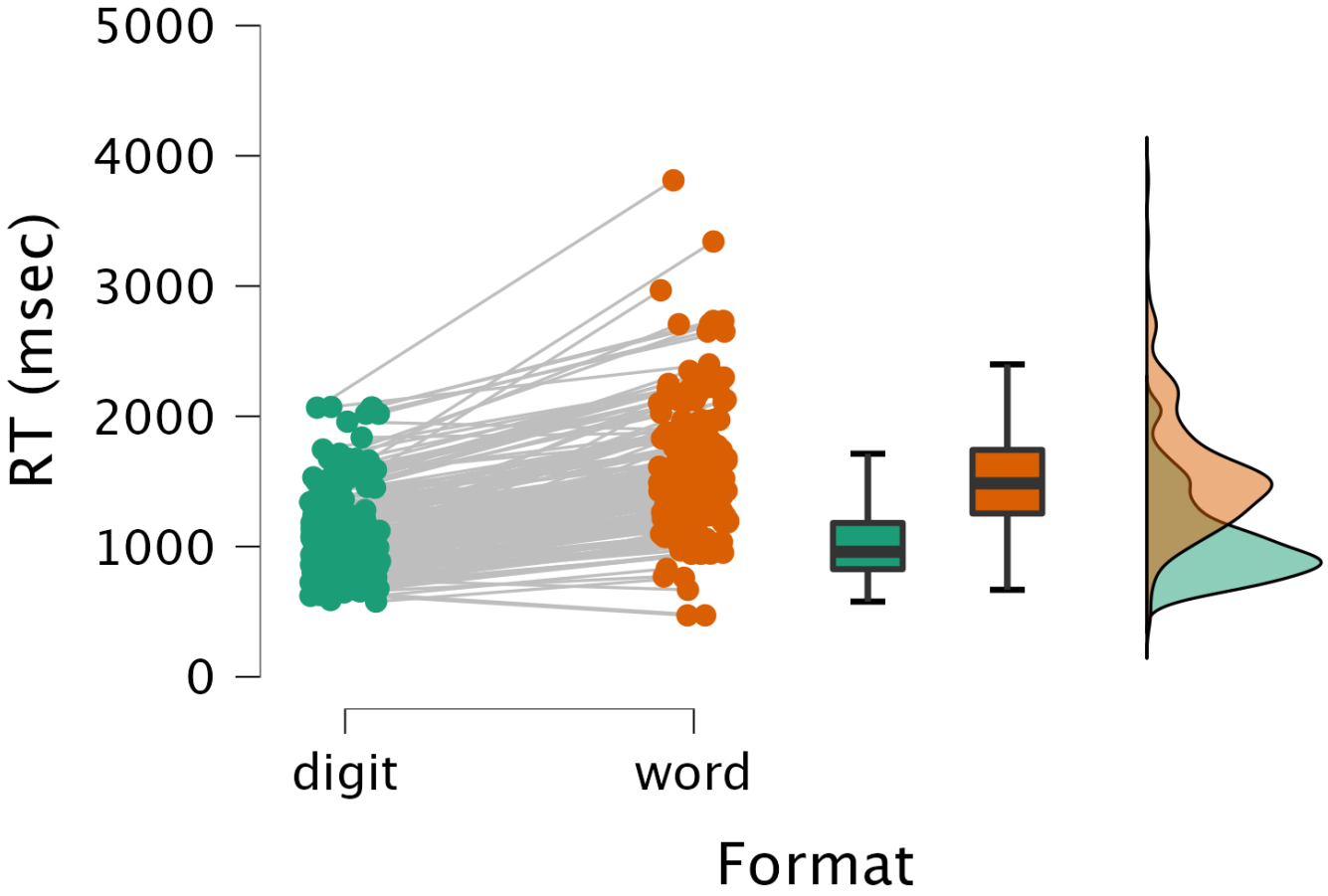
Format effect:



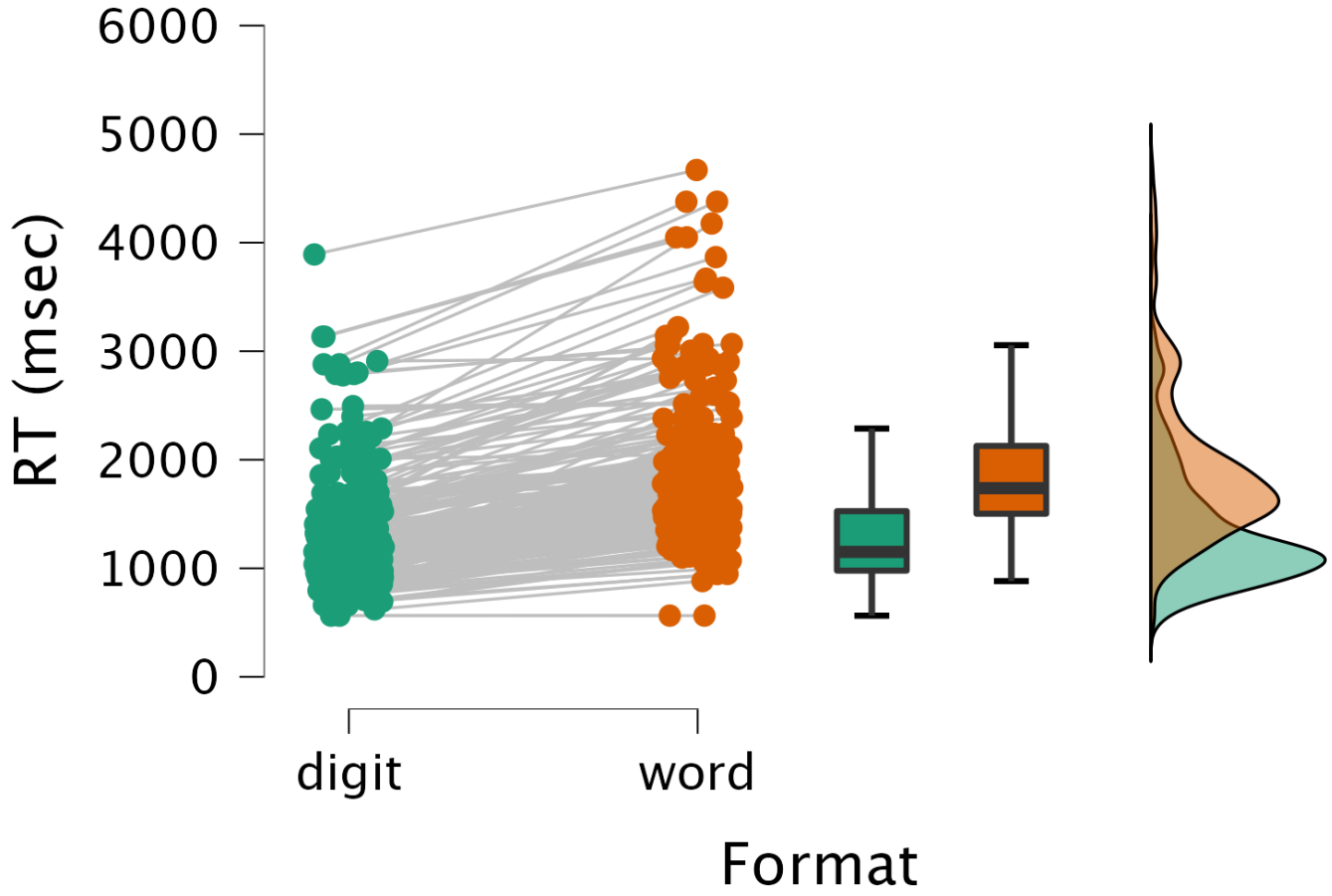
Which RM ANOVA do I use?

- Is it reasonable to assume that there are individual differences in the main effects?
  - Yes!
  - so we use the updated RM ANOVA (van den Bergh et al., 2022)
  - Warning – it takes a long time, especially with $N = 233$!

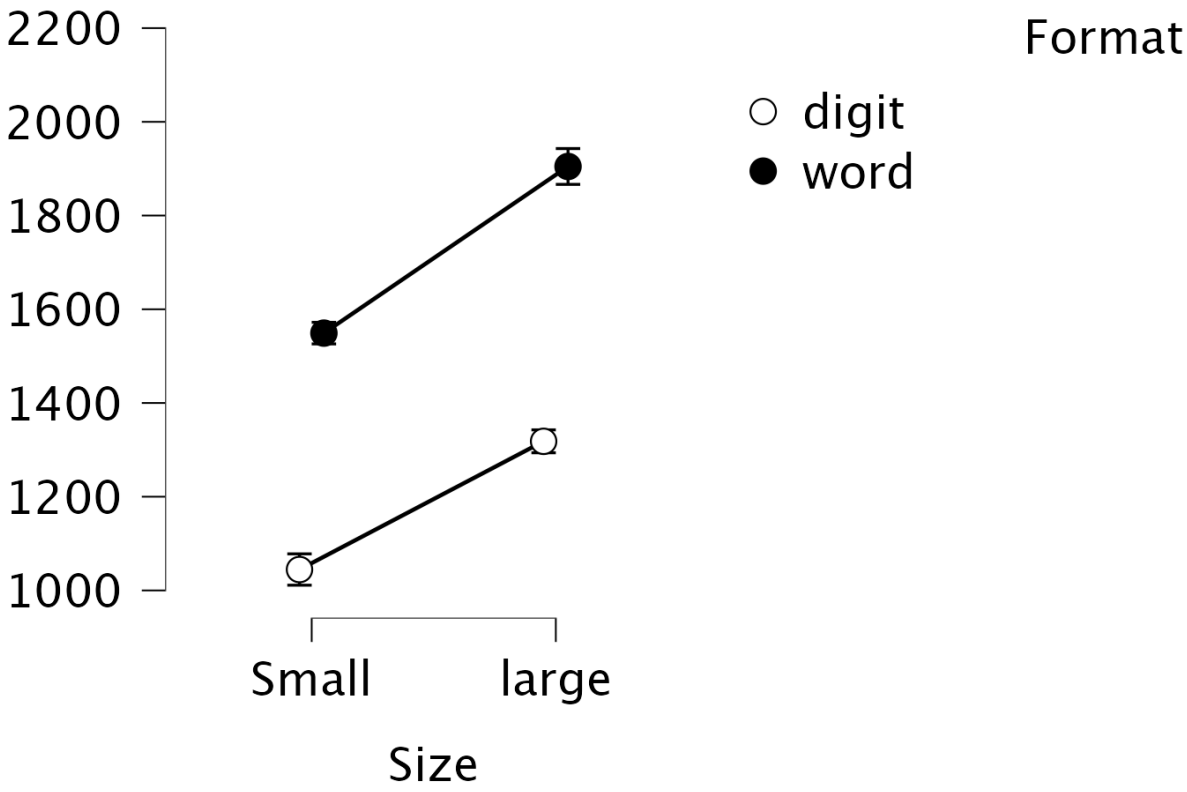
Results – format effects for small problems



Results – format effects for large problems



Results – means



Results – Bayesian model comparison

Models	$\Pr(\mathcal{M})$	$\Pr(\mathcal{M} \mid \text{data})$	$\text{BF}_{\mathcal{M}}$	BF_{10}
size + format + size * format	0.20	0.999	8443	1
size + format	0.20	0.00047	0.002	0.00047
format	0.20	≈ 0	≈ 0	≈ 0
size	0.20	≈ 0	≈ 0	≈ 0
Null model (incl. subject and random slopes)	0.20	≈ 0	≈ 0	≈ 0

The only model which increases model odds is the one containing a size $\times$ format interaction.

Almost all posterior mass is placed on the interaction model.

Results – Bayesian model averaging

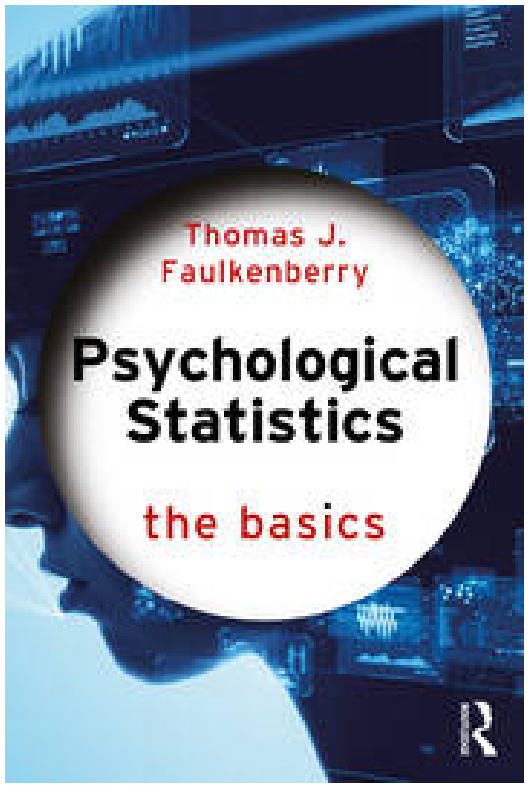
Effects	Pr(inc)	Pr(inc | data)	BF_{inc}
size	0.60	≈ 1	∞
format	0.60	≈ 1	∞
size * format	0.20	0.999	8443

The observed data are 8443 times more likely under models including the interaction term than under models which exclude it.

Take home:

- our data support an interactive model of mental arithmetic, where effects of surface penetrate downstream to directly affect processes in calculation.
- qualitative individual differences in problem size effect
  - data 221 times more likely under model where size effects are $unconstrained$
- quantitative individual differences in format effect
  - data 6.6 times more likely under model where size effects are $strictly\ positive$
- Bayesian RM ANOVA models should include random slopes
  - omitting these slopes attenuates evidence for size $\times$ format interaction (i.e., $\text{BF}_{\text{inc}}(\text{interaction}) < 3$)

Shameless plugs!



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Thank you!

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