Developing an interactive web application for computing Bayes factors from summary statistics

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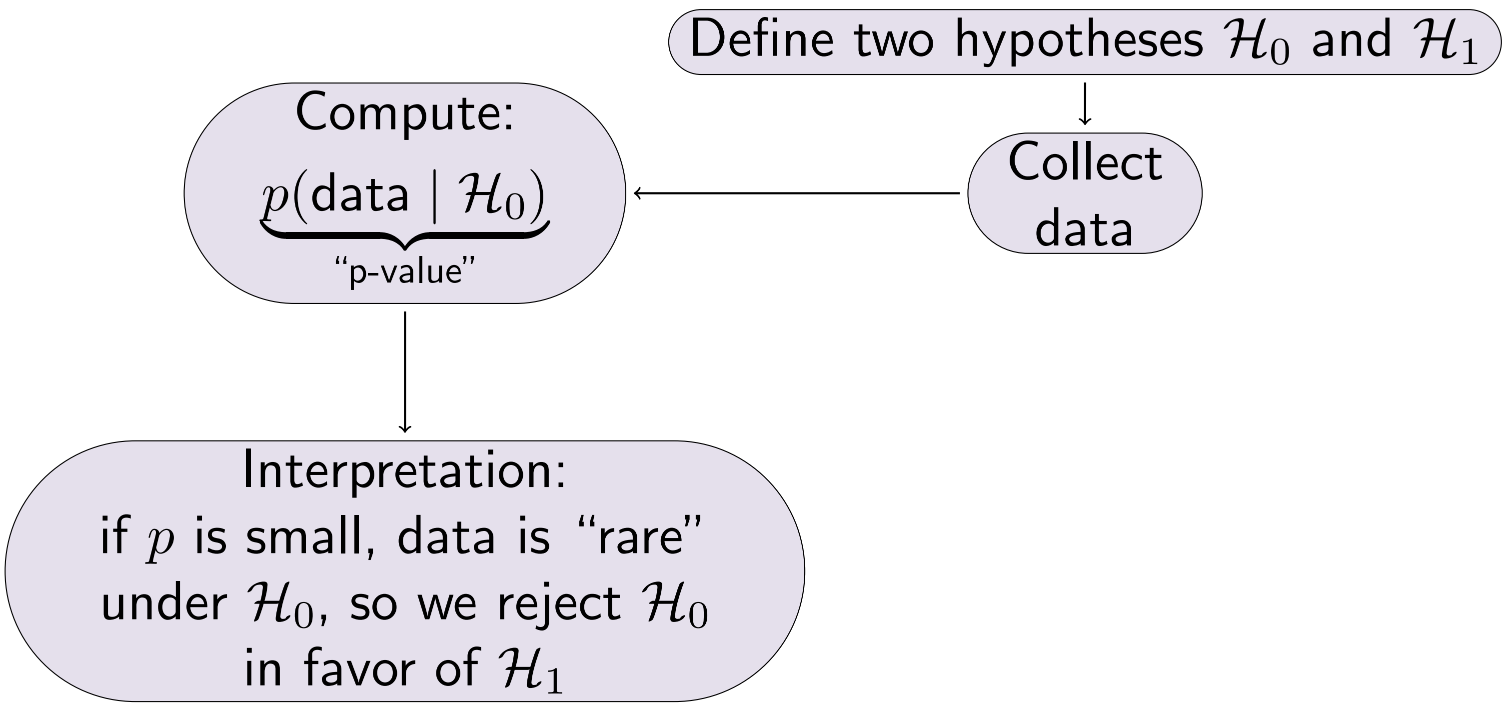
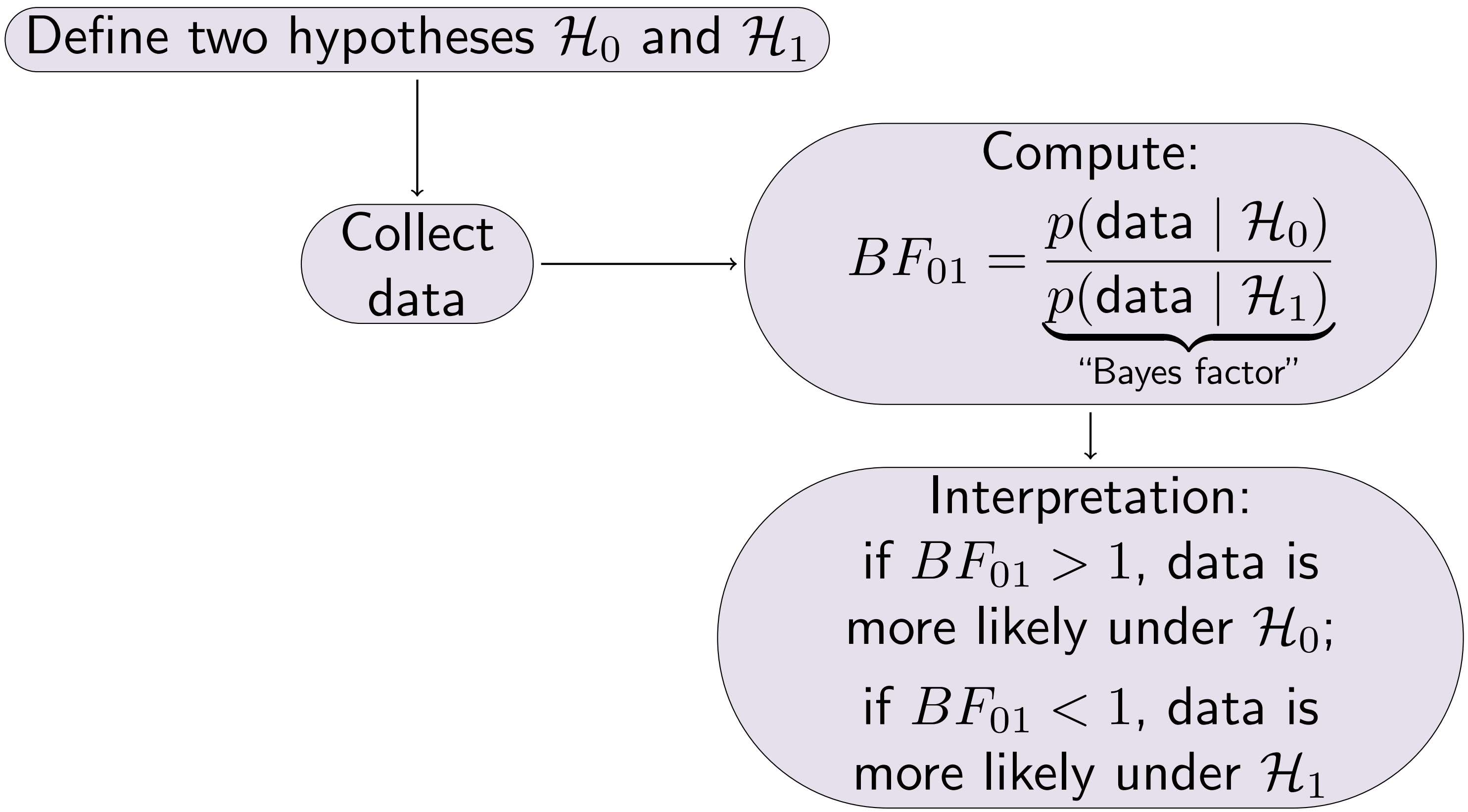
Psychological Sciences Day 2020

Traditional hypothesis testing



Bayesian hypothesis testing



How to compute Bayes factor?

$$BF_{01} = \frac{p(\text{data} \mid \mathcal{H}_0)}{p(\text{data} \mid \mathcal{H}_1)}$$

Suppose simple case: $\mathcal{H}_0 : \mu = 0$, $\mathcal{H}_1 : \mu > 0$

- $p(\text{data} \mid \mathcal{H}_1)$ measures how well $\mathcal{H}_1$ predicts (actually) observed data
- there is uncertainty about the possible values for μ under $\mathcal{H}_1$ – need to place a *prior distribution* on μ to (mathematically) encode this uncertainty
- then we weight the predictive adequacy of each value of μ (under $\mathcal{H}_1$) by the prior – this requires calculus.

To avoid this complexity, Kass and Raftery (1995) introduced the BIC $approximation$, later popularized by Wagenmakers (2007).

Steps:

1. For both models, compute $BIC = n \ln(1 - R^2) + k \ln n$

2. $BF_{01} \approx \exp\left(\frac{BIC(\mathcal{H}_1) - BIC(\mathcal{H}_0)}{2}\right)$

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Computing Bayes factors to measure evidence from experiments: An extension of the BIC approximation

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Summary

Bayesian inference affords scientists powerful tools for testing hypotheses. One of these tools is the Bayes factor, which indexes the extent to which support for one hypothesis over another is updated after seeing the data. Part of the hesitance to adopt this approach may stem from an unfamiliarity with the computational tools necessary for computing Bayes factors. Previous work has shown that closed-form approximations of Bayes factors are relatively easy to obtain for between-groups methods, such as an analysis of variance or t-test. In this paper, I extend this approximation to develop a formula for the Bayes factor that directly uses information that is typically reported for ANOVAs (e.g., the F ratio and degrees of freedom). After giving two examples of its use, I report the results of simulations which show that even with minimal input, this approximate Bayes factor produces similar results to existing software solutions.

Key words: Bayes factors, Bayesian inference, analysis of variance, hypothesis testing

In 2018, I published a paper in *Biometrical Letters* that gave a method for calculating this BIC Bayes factor directy from the summary $F(x,y)$ of an ANOVA:

$$BF_{01} = \sqrt{n^x \left(1 + \frac{Fx}{y}\right)^{-n}}$$

Commentary

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SAGE

Estimating Evidential Value From Analysis of Variance Summaries: A Comment on Ly et al. (2018)

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The past decade has witnessed a tremendous increase in the number of tools that enable social-science researchers to perform Bayesian inference. Ly et al. (2018) recently described one such tool: the Summary Stats module included as part of the open-source software package JASP (JASP Team, 2018). With this tool, researchers can input a minimal set of summary statistics (either from their own previously run analysis or from the published results of other researchers) and obtain a Bayesian reevaluation of the results. In particular, the Summary Stats module reports the Bayes factor (Kass & Raftery, 1995), a continuous index of the extent to which observed data are more likely under one hypothesis than under another, competing hypothesis. For example, the Bayes factor BF_{01} describes the factor by which one's prior belief about the relative likelihood of the null hypothesis H_0 over the alternative hypothesis H_1 should be updated after one observes data. This characterization makes the Bayes factor a useful measure of the *evidential value* of data (Etz & Vandekerckhove, 2017).

As Ly et al. (2018) described so well, the JASP Summary Stats module is a powerful tool that gives the user access to some of the core elements of Bayesian inference without need for accompanying raw data. This provides users with flexibility; researchers may wish to assess the evidential value of their own data to sensibly ground their interpretations, and reviewers or editors may wish to do the same for reported data to help with their own interpretations and decisions regarding publication. However, at present, the Summary Stats module does not include an option for directly reanalyzing the results of an analysis of variance (ANOVA). ANOVA is often characterized as the "workhorse" of experimen-

To this end, I wrote an interactive Web application for calculating Bayes factors from minimal single-factor ANOVA summaries (see Fig. 1). The application, which can accessed by any Web browser at http://tomfaulkenberry.shinyapps.io/anovaBFcalc, performs calculations that are based on the Bayesian information criterion (BIC; Faulkenberry, 2018, 2019; Masson, 2011; Nathoo & Masson, 2016; Wagenmakers, 2007). It requires minimal input; the user need only specify the F statistic and the degrees of freedom for the ANOVA. Additionally, the user may specify the design (between subjects or repeated measures) and a prior probability for H_0 (default = .5). In return, the application provides the user with estimates of BF_{01} and the reciprocal BF_{10} (i.e., $1/BF_{01}$) a sentence interpreting what the estimate of the Bayes factor for the "winning" model means, a graphical display of the strength of evidence indicated by the Bayes factor (i.e., a *pizza plot*;[1] Wagenmakers et al., 2017), and an estimate of the posterior probability of each hypothesis.

Disclosures

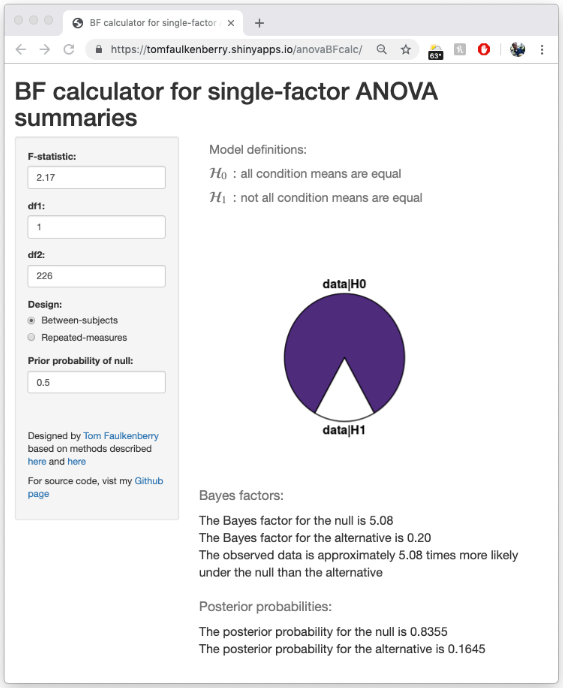
The source code for the interactive Bayes factor calculator described in this article can be accessed at https://github.com/tomfaulkenberry/anovaBFcalc.

Example

For an illustration of the use of the online calculator, consider the following results from Rovenpor et al. (2019). In several experiments, Rovenpor et al. investigated whether violent conflict provides people with an enduring sense of meaning, thus perpetuating further

Last summer, I created an interactive web application for computing this Bayes factor – its basic functionality is described in a paper in *Advances in Methods and Practices in the Psychological Sciences* (AMPPS)

http://tomfaulkenberry.shinyapps.io/anovaBFcalc



Why not just use JASP?

- the app is platform independent (any browser, even on smartphone!)
- the app requires no software installation
- no need to maintain a computer lab

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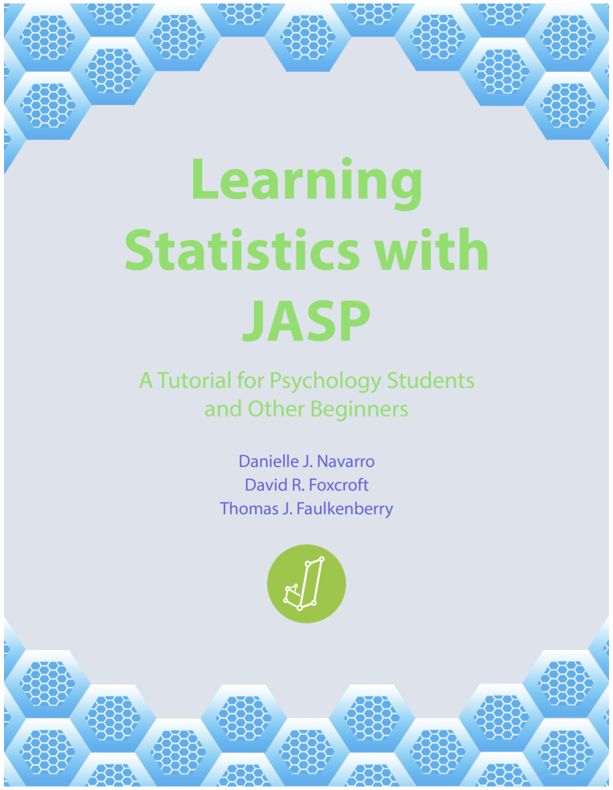
Current limitations:

- it uses an $approximation$ to the Bayes factor
- only works for ANOVA summaries
- the code-base is not particularly robust

Plans for next year:

1. extend to repeated-measures designs (done – https://arxiv.org/abs/1905.05569)
2. develop closed-form Bayes factors for ANOVA summary statistics (preprint coming soon)
3. extend the web application to include t-tests and correlations (planned)
4. re-write the web application to a fully-fledged R package (started)
5. develop rudimentary Bayesian "power analysis" module (started)

Highlight of the year: in Summer 2019, I wrote an open-source textbook (www.learnstatswithjasp.com)



This led to an invitation to visit E.J. Wagenmakers and work with the JASP team at the University of Amsterdam for a week in October 2019:

..which led to a new Bayes tutorial paper (https://psyarxiv.com/vg9pw), currently under review.

Bayesian Inference in Numerical Cognition: A Tutorial Using JASP

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Abstract

Researchers in numerical cognition rely on hypothesis testing and parameter estimation to evaluate the evidential value of data. Though there has been increased interest in Bayesian statistics as an alternative to the classical, frequentist approach to hypothesis testing, many researchers remain hesitant to change their methods of inference. In this tutorial, we provide a concise introduction to Bayesian hypothesis testing and parameter estimation in the context of numerical cognition. Here, we focus on three examples of Bayesian inference: the *t*-test, linear regression, and analysis of variance. Using the free software package JASP, we provide the reader with a basic understanding of how Bayesian inference works "under the hood" as well as instructions detailing how to perform and interpret each Bayesian analysis.